Computers and Chemical Engineering 71 (2014) 347–361

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Game-theoretic modeling and optimization of multi-echelon supply chain design and operation under Stackelberg game and market equilibrium

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ARTICLE INFO

Article history:
Received 4 June 2014
Received in revised form 20 August 2014
Accepted 31 August 2014
Available online 16 September 2014

Keywords:
Supply chain optimization
Game theory
Stackelberg game
Generalized Nash equilibrium
Improved branch-and-refine algorithm
Biofuel supply chain

ABSTRACT

We propose a bilevel mixed-integer nonlinear programming (MINLP) model for the optimal design and planning of non-cooperative supply chains from the manufacturer's perspective. Interactions among the supply chain participants are captured through a single-leader–multiple-follower Stackelberg game under the generalized Nash equilibrium assumption. Given a three-echelon superstructure, the lead manufacturer in the middle echelon first optimizes its design and operational decisions, including facility location, sizing, and technology selection, material input/output and price setting. The following suppliers and customers in the upstream and downstream then optimize their transactions with the manufacturer to maximize their individual profits. By replacing the lower level linear programs with their KKT conditions, we transform the bilevel MINLP into a single-level nonconvex MINLP, which is further globally optimized using an improved branch-and-refine algorithm. We also present two case studies, including a county-level biofuel supply chain in Illinois, to illustrate the application of the proposed modeling and solution methods.

1. Introduction

When entering a business, a manufacturer is often encountered with questions such as where to locate the plants, what sizes the plants should be, which conversion technology to choose, how much to produce, and how to set the transfer prices (Grossmann, 2005). Although there is a large body of literature on the modeling and optimization of supply chain design and operations, most of these works view the supply chain from a centralized perspective and integrate the various components of the supply chain into a monolithic model (Muñoz et al., 2013; Papageorgiou, 2009; Shah, 2005). Under this approach, it was implicitly assumed that the decision maker has full control over the entire supply chain so that all the strategic and operational decisions can be implemented successfully. However, the management over a supply chain is often decentralized in practice (Cachon and Netessine, 2004). In other words, different stakeholders may be in charge of different entities in the supply chain, and these stakeholders may even have conflicting interests against each other. These supply chain participants would strive to maximize their own benefits and compete with their peers, thus leading to a non-cooperative supply chain (Facchinei and Kanzow, 2007).

The goal of this work is to develop a novel game-theoretic modeling and optimization framework that addresses non-cooperative supply chain design and planning from a manufacturer's perspective. In a non-cooperative supply chain, all of the participants act selfishly and are solely driven by their own objective. The players make decisions independently without collaboration or communication (Nash, 1951). This is different from a cooperative supply chain, where the players are willing to negotiate with each other and arrive at a unanimous agreement (Nagarajan and Sošić, 2008). Recent works on cooperative supply chains include the works by Gjerdrum et al. (2001, 2002), Zhang et al. (2013), Fernandes et al. (2013), Banaszewski et al. (2013), and Yue and You (2014). On the other hand, representative works on non-cooperative supply chains include the works by Bard et al. (2000), Ryu et al. (2004), and Zamarripa et al. (2012, 2013). To the best of our knowledge, most existing works on the optimal design and planning of non-cooperative supply chains are restricted to a rather simple supply chain structure, instead of the multi-echelon network considered in this work. Furthermore, in previous works, linearization assumptions and simplifications have been applied in order to keep the model tractable, which might lose the generality of the mathematical model. Therefore, this work aims to fill these research

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http://dx.doi.org/10.1016/j.compchemeng.2014.08.010

Nomenclature

Sets/indexes

b	biomass types
i	suppliers
j	biorefineries
k	customers
p	biofuel types
q	conversion technologies

Parameters

$c^0_{j,q}$	reference capital cost of technology q at biorefinery j
$c^{L(U)}_{j,q}$	minimum (maximum) capacity limit of biorefinery j with technology q
$ca_{b,i}$	harvesting cost of biomass b at supplier i
$cap^0_{j,q}$	reference capacity of technology q at biorefinery j
cf_q	fixed O&M cost of technology q as a percentage of the capital cost
$cij_{b,i,j}$	transportation cost of biomass b from supplier i to biorefinery j
$cjk_{p,j,k}$	transportation cost of biofuel p from biorefinery j to customer k
$cp_{j,q}$	variable production cost at biorefinery j with technology q
$d^U_{p,k}$	upper bound of the demand for biofuel p at customer k
ir	discount rate
lt	lifetime of the project
num_q	maximum number of biorefineries with technology q allowed to be installed
$pr_{p,k}$	market price of biofuel p at customer k
prb_b	price of per unit of biomass b offered by external market
prp_p	price of per unit of biofuel p offered by external market
sf	scaling factor
$wi^U_{b,j,q}$	upper bound of consumption of biomass b at biorefinery j via technology q
$wo^U_{p,j,q}$	upper bound of production of biofuel p at biorefinery j via technology q

Greek parameters

$\theta_{j,q}$	minimum utilization rate of biorefinery j with technology q
φ_p	common functional unit corresponding to a unit of biofuel p
$\chi_{b,p,j,q}$	conversion parameter between biomass b and biofuel p at biorefinery j with technology q

Non-negative variables

C^{cap}	amortized capital cost
C^{om}	annual operation and maintenance cost
C^{ex-aq}	cost of biomass acquisition from the external market
C^{ex-sa}	revenue of biofuel sales to the external market
$Cap_{j,q}$	capacity of biorefinery j with technology q
$Fij_{b,i,j}$	amount of biomass b shipped from supplier i to biorefinery j
$Fjk_{p,j,k}$	amount of biofuel p shipped from biorefinery j to customer k
$Pa_{b,j}$	acquisition price of biomass b at biorefinery j
$Ps_{p,j}$	selling price of biofuel p at biorefinery j
$Wi_{b,j}$	acquisition target of biomass b at biorefinery j
$Wiq_{b,j,q}$	amount of biomass b to consume at biorefinery j via technology q
$Wo_{p,j}$	production target of biofuel p at biorefinery j
$Woq_{p,j,q}$	amount of biofuel p to produce at biorefinery j via technology q

Binary variables

$X_{j,q}$	1 if biorefinery j with technology q is installed; otherwise 0

gaps by both proposing a novel *single-leader–multiple-follower* game-theoretic framework for general three-echelon supply chain networks and developing an effective global optimization strategy to address the resulting mathematical model.

Specifically, we consider a three-echelon supply chain superstructure, which includes a set of candidate sites for building manufacturing facilities in the middle echelon, as well as a set of given upstream suppliers and downstream customers. A single player – a manufacturer – is in charge of all the manufacturing facilities, while each supplier and customer is considered to be an independent player. The manufacturer is assumed to be the supply chain leader, who makes the decisions on supply chain design and strategic planning first. The suppliers and customers are assumed to be the followers who then maximize their own profits given the leader's decisions. We model the leader–follower relationship as a *single-leader–multiple-follower* Stackelberg game. We model the competition among suppliers and customers under the assumption of generalized Nash equilibrium.

Without loss of generality, we formulate the problem above into a bilevel mixed-integer nonlinear program (MINLP). In the upper level problem, the manufacturer optimally determines the location, capacity, and technology of the manufacturing facilities, as well as the operational plans and transfer prices in order to maximize the total profit generated from all the manufacturing facilities. Discrete variables are employed to select among the candidate sites, conversion technologies, etc. Capital cost economies of scale are captured by using a nonlinear power function. To calculate the transfer payments of materials, bilinear terms are also included. Therefore, the upper level problem is nonlinear, nonconvex, and has combinatorial features. Given the manufacturer's decisions in the upper level problem, each follower in the lower level problem optimizes its transactions with the installed manufacturing facilities to maximize its own profit. Specifically, the suppliers optimize the amount of raw materials to be sold to the manufacturing facilities and the customers optimize the amount of products to purchase from the manufacturing facilities. All of the lower level problems are formulated as linear programs (LPs).

We note that the resulting bilevel program cannot be handled directly using off-the-shelf optimization solvers. However, for the cases that all the lower level problems are LPs, we can reformulate the bilevel MINLP problem into an equivalent single-level MINLP by replacing each follower's optimization problem with the corresponding Karush–Kuhn–Tucker (KKT) conditions (Bard, 1998). Though solvable, the resulting single-level MINLP problem can still be computationally intractable due to the presence of concave and bilinear terms as well as integer variables. To facilitate the solution process, we further propose an improved branch-and-refine algorithm which is based on a class of SOS1 (specially ordered set of type 1) piecewise linear formulations. The algorithm takes advantage of powerful mixed-integer linear programming (MILP) solvers and globally optimizes the nonconvex MINLP problem efficiently in finite iterations. To illustrate the application of the proposed