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On-line computation of Stackelberg equilibria with synchronous parallel genetic algorithms

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Abstract

This paper develops a method to compute the Stackelberg equilibria in sequential games. We construct a normal form game which is interactively played by an artificially intelligent leader, GA^L, and a follower, GA^F. The leader is a genetic algorithm breeding a population of potential *actions* to better anticipate the follower's reaction. The follower is also a genetic algorithm training on-line a suitable neural network to evolve a population of *rules* to respond to any move in the leader's action space. When GAs repeatedly play this game updating each other synchronously, populations converge to the Stackelberg equilibrium of the sequential game. We provide numerical examples attesting to the efficiency of the algorithm. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

In a sequential game, the player who has the first move advantage is the natural leader. If players' costs are common knowledge, the leader can fully anticipate the follower's response to any move in her action space. Therefore, she will act so as to elicit the most favorable response from the follower. Analytically, the leader's problem is tantamount to a cost minimization constrained by the follower's reaction function. The resulting equilibrium is also known as the Stackelberg equilibrium.

The Stackelberg equilibrium concept requires that the leader have the capacity to fully anticipate the follower's reactions to each and every move in her action space.

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This is indeed a strong form of rationality. An interesting question in this regard is whether boundedly rational players can *learn* to play the Stackelberg equilibrium if they have the opportunity to play repeatedly. That is, whether the Stackelberg equilibrium of a one-shot sequential game where players are perfectly rational can be generated as the equilibrium of a game where boundedly rational players start ignorant, but learn as the game repeats. In this paper, our answer to this inquiry is affirmative provided that the leader learns to act by discretion while the follower learns to play by the rule.

Towards that, we formulate a normal form game in which the leader's strategy space consists of a set of actions while the follower's strategy space is comprised of a set of rules. We then parameterize the follower's strategy space by the weights of a suitable feed-forward neural network, thereby transforming it from a set of rules to a set of neural net weights.

Two artificially intelligent players, GA^F and GA^L, play the normal form game generation after generation. The follower population evolves the weights of a given feed-forward neural network to come up with a best response to the leader population's best action in the previous generation. As the search progresses in the leader's strategy space, the follower population is *trained* to best respond to any action of the leader. The fittest individuals in the respective populations are then communicated to each other via the computer shared memory. Equipped with the updated weights, the leader is better able to anticipate the best response of the follower for all potential actions in its population. The individuals which exploit this knowledge to their advantage are fitter, consequently, they reproduce faster. Ultimately, they dominate the leader population also steering the follower's search for the best set of rules in vicinity of the Stackelberg solution.

Our method has computational advantages as well. We show that on-line synchronous parallel genetic algorithms can compute the Stackelberg equilibria with efficiency. In our approach, the neural net is trained not over the entire strategy space of the leader at once, but rather incrementally as it responds to any given action of the leader in the course of the repeated game. This is important, because both players, GA^F and GA^L start the game completely blind, but learn as the game unfolds.

It is worth emphasizing that our approach does not require any knowledge of the follower's reaction function. The neural network parameterization provides with a high level of flexibility and can be used for problems in which the follower's reaction function cannot be analytically obtained. Thus, the computational effort and time required by an off-line algorithm is considerably reduced. As a drawback, we can cite less reliability of the follower's training since it is on-line.

Li and Başar (1987) show that iterative on-line asynchronous algorithms converge to the Nash equilibrium if players' costs are private information. Vallée and Başar (1999) use an *off-line* genetic algorithm GA to compute the Stackelberg equilibria. For each action in the leader population, they compute the follower's best response off-line and then feed it back to the leader GA. The GA converges to the Stackelberg equilibrium. This method, however, is computationally too intensive as one needs to go off-line every time the follower's best response is needed to evaluate the fitness of an individual in the leader population.