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A note on dynamic Stackelberg games with leaders in turn

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ABSTRACT

Recently, a model of dynamic Stackelberg games with leaders in turn has been proposed, and dynamic Stackelberg games with leaders in turn have been exploited under a feedback information structure. This paper characterizes dynamic Stackelberg games with leaders in turn under other information structures, both closed-loop and open-loop information structures. Explicit solutions are given for linear-quadratic systems under an open-loop information structure for dynamic Stackelberg games with leaders in turn.

1. Introduction

In many decision processes there exists a hierarchy of decision makers and these processes are modelled as a bilevel programming problem in economics and management fields [1,2]. The static bilevel model was initially introduced by Von Stackelberg [3] and there exists extensive research on bilevel optimization [4,5] in the optimization community. Studies on dynamic bilevel optimization seem relatively scarce because of its challenges. Dynamic bilevel optimization was first introduced by Chen and Cruz [6], and it has subsequently been studied by a number of authors [7–10].

In almost all the literature about dynamic Stackelberg games, the positions of both the leader(s) and the followers are always assumed to be fixed during the game. But in many practical games, there exist leaders in turn. To model overconfident consumers, for example, Grubb [11] established a two-stage game theory model. In the first stage, a firm offers a tariff menu and consumers accept or reject it. This is a principal-agent game or a Stackelberg game, in which the firms play the leading positions while the consumers act as followers. In the second stage, consumers gather their own information, purchase the required quantity, and pay the bill. In this stage, the consumers play the leading positions while the firms act as followers. This example also appeared in [12]. Başar et al. [13] also gave an interesting example with an unfixed leader, which includes a manufacturer and a retailer. The manufacturer acts as the leader for advertising decisions. The retailer acts as the price leader. All these interesting phenomena motivate this work.

Nie [2] and Nie et al. [14] recently proposed dynamic Stackelberg game models with unfixed leaders. At the same time, Başar et al. [13] proposed this type of differential game, called mixed-leadership games. The open-loop situation was highlighted by Başar et al. [13]. In this paper, we reconsider discrete-time dynamic Stackelberg games with leaders in turn under feedback, closed-loop, and open-loop information structures. For the games described in this paper, all players are rational in the decision process.

Discrete-time dynamic Stackelberg games, with feedback information and leaders in turn, are described in the following; see also [2]. The discrete time periods are denoted $t = 1, 2, \ldots, T$, and N is the number of players.

The variables involved in the problem are listed as follows.

Vectors $x_t^\kappa \in \mathcal{X} \subset R^{m^\kappa}$ denote the state of player κ at time $t = 1, 2, \ldots, T$ for $\kappa = 1, 2, \ldots, N$.

Vectors $u_t^\kappa \in \mathcal{U} \subset R^{n^\kappa}$ denote the decision variables for player κ at time $t = 1, 2, \ldots, T - 1$ for $\kappa = 1, 2, \ldots, N$.

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Moreover, for $t = 1, 2, \ldots, T-1$ and $\kappa = 1, 2, \ldots, N$, we denote

$$\begin{aligned}
&x_t^{-\kappa} := (x_t^1, x_t^2, \ldots, x_t^{\kappa-1}, x_t^{\kappa+1}, \ldots, x_t^N),\\
&u_t^{-\kappa} := (u_t^1, u_t^2, \ldots, u_t^{\kappa-1}, u_t^{\kappa+1}, \ldots, u_t^N),\\
&x_t := (x_t^1, x_t^2, \ldots, x_t^N),\\
&x := (x_1, x_2, \ldots, x_T) = (x_t^\kappa, x_t^{-\kappa}),\\
&u_t := (u_t^1, u_t^2, \ldots, u_t^N),\\
&u := (u_1, u_2, \ldots, u_{T-1}) = (u_t^\kappa, u_t^{-\kappa})\\
&x_{n_1,n_2} := (x_{n_1}, x_{n_1+1}, \ldots, x_{n_2}) = (x_{n_1,n_2}^\kappa, x_{n_1,n_2}^{-\kappa}),\\
&u_{n_1,n_2} := (u_{n_1}, u_{n_1+1}, \ldots, u_{n_2}) = (u_{n_1,n_2}^\kappa, u_{n_1,n_2}^{-\kappa}),\\
&\text{for any } 1 \le n_1 < n_2 \le T-1,\ \kappa = 1, 2, \ldots, N.
\end{aligned}$$

The state variables $\{x_t\}_{t=1}^T$ are governed by the system of state transition equations

$$x_{t+1}^\kappa = F_t^\kappa(x_t^\kappa, u_t^\kappa), \quad t = 1, 2, \ldots, T-1,\ \kappa = 1, 2, \ldots, N, \tag{1.1}$$

with x_1 being given as the initial states of players.

We denote the feasible domain of player κ to be

$$\Pi_t^\kappa(x_t^\kappa) := \left\{u_t^\kappa | h_t^\kappa(x_t^\kappa, u_t^\kappa) \le 0\right\},$$

where $h_t^\kappa : R^{m^\kappa} \times R^{n^\kappa} \to R^{n^\kappa}$ is a given function, and $u_t^\kappa \in \Pi_t^\kappa(x_t^\kappa) \subset \mathcal{U} \subset R^{n^\kappa}$. Let the set of admissible decisions of player κ be given by

$$\Pi^\kappa(x^\kappa) := \left\{u^\kappa | h_t^\kappa(x_t^\kappa, u_t^\kappa) \le 0,\ t = 1, 2, \ldots, T-1\right\}.$$

For convenience, for $\kappa = 1, 2, \ldots, N$, the following notation will be frequently employed in this paper:

$$\begin{aligned}
&F_t^{-\kappa} := (F_t^1, F_t^2, \ldots, F_t^{\kappa-1}, F_t^{\kappa+1}, \ldots, F_t^N),\\
&F_t := (F_t^\kappa, F_t^{-\kappa}) = (F_t^1, F_t^2, \ldots, F_t^N),\\
&\Pi_t^{-\kappa} := \Pi_t^1 \times \Pi_t^2 \times \cdots \times \Pi_t^{\kappa-1} \times \Pi_t^{\kappa+1} \cdots \times \Pi_t^N,\\
&\Pi_{t,T-1}^\kappa(x_{t,T-1}^\kappa) := \left\{u_{t,T-1}^\kappa | h_\tau^\kappa(x_\tau^\kappa, u_\tau^\kappa) \le 0,\ \tau = t, t+1, \ldots, T-1\right\}.
\end{aligned}$$

Moreover, for $\kappa = 1, 2, \ldots, N, J_t^\kappa$, which is the total cost function after stage t, is defined as follows:

$$J_t^\kappa(x_t, u_{t,T-1}) := \sum_{\tau=t}^{T-1} g_\tau^\kappa(x_\tau, u_\tau) + g_T^\kappa(x_T), \quad \kappa = 1, 2, \ldots, N,\ t = 1, 2, \ldots, T,$$

where g_t^κ is the cost function for player κ at stage t and the $\{x_t\}_{t=0}^T$ are determined by (1.1).

The problem is formally stated as follows, assuming the first player to be the initial leader and given the initial state $x_1 = a_1 \in \mathcal{X}$. If $t \equiv \kappa \pmod N$, where $1 \le \kappa \le N$ (we say that $t \equiv \kappa \pmod N$ if and only if there exists some integer i such that $t = \kappa + Ni$), then, consider the following problems, which are referred to as $P_{T-t}^\kappa(x_t)$ for $t = 1, 2, \ldots, T-2$.

$$\begin{array}{ll}
\min\limits_{u_t^\kappa} & J_t^\kappa(x_t, u_{t,T-1})\\
\text{s. t} & u_t^\nu \in \arg\min\{J_t^\nu(x_t, u_{t,T-1})\},\\
& u_{t,T-1}^\nu \in \Pi_{t,T-1}^\nu(x_{t,T-1}^\nu),\\
& u_{t,T-1}^\kappa \in \Pi_{t,T-1}^\kappa(x_{t,T-1}^\kappa), \quad \nu \ne \kappa,\\
& u_{t+1,T-1} \text{ solves } P_{T-t-1}^{\hat\kappa}(x_{t+1}),
\end{array} \tag{1.2}$$

where $\hat\kappa = \kappa + 1$ if $\kappa < N$. Otherwise, $\hat\kappa = 1$. For $t = T-1$, we give $P_{T-(T-1)}^\kappa$ as follows

$$\begin{array}{ll}
\min\limits_{u_{T-1}^\kappa} & J_{T-1}^\kappa(x_{T-1}, u_{T-1})\\
\text{s.t} & u_{T-1}^\nu \in \arg\min\{J_{T-1}^\nu(x_{T-1}, u_{T-1})\},\\
& u_{T-1}^\nu \in \Pi_{T-1}^\nu(x_{T-1}^\nu),\\
& u_{T-1}^\kappa \in \Pi_{T-1}^\kappa(x_{T-1}^\kappa), \quad \nu \ne \kappa,
\end{array} \tag{1.3}$$

where $\kappa \equiv T-1 \pmod N$ and $1 \le \kappa \le N$. We point out that subproblem $P_{T-t-1}^{\hat\kappa}(x_{t+1})$ can be rewritten as $P_{T-t-1}^{\hat\kappa}(F_t(x_t, u_t))$, in which $P_{T-t-1}^{\hat\kappa}(x_{t+1})$ is determined by x_t and u_t.

Problem P_{T-1}^1 is referred to as a dynamic bilevel optimization problem with leaders in turn under feedback information structure, or a DBOPLT for short. We point out that the DBOPLT consists of a sequence of minimization problems.

For $\kappa = 1, 2, \ldots, N$, if

$$x_{t+1}^\kappa = F_t^\kappa(x_0, u_t^\kappa) \quad t = 0, 1, \ldots, T-1, \tag{1.4}$$