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Dynamic Stackelberg games under open-loop complete information

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Abstract

Stackelberg games, which play extremely important roles in such fields as economics, management, politics and behavioral sciences, can be modelled as a bilevel optimization problem. There exist extensive literatures about static bilevel optimization problems. However, the studies on dynamic bilevel optimization problems are rather scarce in spite of the importance in explaining and predicting some phenomena rationally. In this paper, discrete time dynamic Stackelberg games with open loop complete state are revisited. An example, which comes from law field, is presented as an open-loop Stackelberg game to illuminate the rangy application of game theory. A property based on Bellman's equation is proposed without any restriction about inner point solution for open-loop Stackelberg games in this work. Moreover, we point out that open-loop Stackelberg games perform no better than both feedback and closed-loop dynamic Stackelberg games with complete information, which seems quite valuable to explain some social and economic phenomena.

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Keywords: Discrete time dynamic models; Stackelberg games; Open-loop complete information; Law; Enforcement

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1. Introduction

In many decision processes there is a hierarchy of decision makers. Decisions are made at different levels with different goals in this hierarchy. Moreover, those decision makers often cannot act independently of each other but have to take into account decisions made by players of different levels. Specially, in a bilevel case, the optimal strategies chosen by the lower level players (hereafter the "followers") depend on the strategy selected by the upper level player (hereafter the "leader"). On the other hand, the objective function of the leader may depend not only on his/her own decisions but also on the followers'. The leader is then able to make his/her decisions by estimating the followers' rational reactions, assuming that the followers optimize their objective functions given the leader's actions. This is the static bilevel model introduced by Von Stackelberg [1]. There exists extensive research on bilevel optimization [2]. Under certain conditions, a bilevel optimization problem can be reformulated as a mathematical program with equilibrium constraints (MPEC) problem [3], which has recently drawn much attention in the optimization community [4–6]. When players interact by playing a similar stage game numerous times, the game is called a dynamic, or repeated game. Unlike static games, players have at least some information about the strategies chosen on others and thus may contingent their play on past moves. However, the study on dynamic bilevel optimization is relatively scarce. Dynamic bilevel optimization was first introduced by Chen and Cruz [7] and there are some subsequent researches in [8–11]. Particularly, Ye [10,11] recently studies continuous time dynamic bilevel problems.

The discrete time dynamic optimization problem has comprehensive applications in economics and management sciences and has been studied extensively [12]. There exist varied discrete time dynamic games according to information structure, such as open-loop, feedback and closed-loop dynamic Stackelberg games. In this paper, we consider a discrete time dynamic bilevel optimization problem with open-loop complete information, where the state of the state at the initial and the current decision determined the state of the subsequent state.

Let us give the formal state of the problem. The discrete time period is denoted $t = 0, 1, \ldots, T$ and N is the number of the followers in the game.

The variables involved in the problem are listed as follows: Vectors $x_t \in \mathcal{X} \subset R^{m_0}$ denote the state of the leader for $t = 0, 1, \ldots, T$. Vectors $y_t^v \in \mathcal{Y}^v \subset R^{m_v}$ denote the state of the vth follower at time $t = 0, 1, \ldots, T$. The followers' state variables at time t are collectively denoted vectors

$$y_t := (y_t^1, y_t^2, \ldots, y_t^N) \in \mathcal{Y} \subset R^m$$

with $m = m_1 + m_2 + \cdots + m_N$.

Vectors $u_t \in \mathcal{U} \subset R^{n_0}$ denote the decision variables to be selected by the leader at time $t = 0, 1, \ldots, T-1$. Vectors $v_t^v \in \mathcal{V}^v \subset R^{n_v}$ are the decision variables to be selected by the followers at time $t = 0, 1, \ldots, T-1$. The followers' decision variables