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On the endogeneity of Cournot, Bertrand, and Stackelberg competition in oligopolies☆

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ABSTRACT

In many industries, firms pre-order input and forward sell output prior to the actual production period. It is known that forward buying input induces a "Cournot–Stackelberg endogeneity" (both Cournot and Stackelberg outcomes may result in equilibrium) and forward selling output induces a convergence to the Bertrand solution. I analyze the generalized model where firms pre-order input and forward sell output. First, I consider oligopolists producing homogenous goods, generalize the Cournot–Stackelberg endogeneity to oligopoly, and show that it additionally includes Bertrand in the generalized model. This shows that the "mode of competition" between firms may be entirely endogenous. Second, I consider duopolies producing heterogenous goods. The set of equilibrium outcomes is characterized and shown not to contain the Bertrand solution anymore. Yet, forward sales increase welfare also in this case, notably even when goods are complements.

1. Introduction

The paper analyzes oligopolistic industries in a model that explicitly contains planning periods prior the production period. The model sets in T periods prior to the production period, e.g. $T=52$ weeks prior to the year 2012. In these T preliminary stages, firms may pre-order input (i.e. pre-build capacity for 2012) and conclude forward contracts to sell the output that they will produce. In the production period, firms set production quantities and sell the output that was not sold via forward contracts. Capacities can be extended in the production period, possibly at incremental costs.

This model unifies two streams of literature—the studies of production timing following Saloner (1987) and those of sales timing following Allaz and Vila (1993). I consider production timing in the sense of capacity pre-building, e.g. pre-ordering machinery or raw materials, and allow for sales timing via efficient forward contracts (e.g. to retailers). In many markets, sales and production timing interact, but their interaction has not yet been analyzed and it is therefore unclear whether sales or production timing dominates from a strategic point of view.

If production timing dominates, then the "Cournot–Stackelberg endogeneity" derived by Saloner (1987), Pal (1991), and Romano and Yildirim (2005) results. Their studies show that in two-stage games of (quantity) accumulation, a continuum of outcomes may result in equilibrium that contains both Stackelberg outcomes and the Cournot outcome. However, this continuum results only if the costs of production do not change between first and second periods, and it is established only for the case that quantity produced in stage 1 cannot be withheld from being sold in the final stage (stage 2). The latter assumption is relaxed in the model of "capacity accumulation" analyzed here,[1] and interestingly the Cournot–Stackelberg endogeneity turns out to be generic in case "capacity" is accumulated: The mode of competition is endogenous whenever long-term capacity extensions are not more costly than short-term ones. This kind of endogeneity shows that industries need not converge to Cournot equilibrium, that non-cost-related size differences may persist in

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[1] Following Kreps and Scheinkman (1983), Saloner (1987), and many subsequent studies on capacity-constrained price competition (Allen et al., 2000; Boccard and Wauthy, 2000, 2010; Reynolds and Wilson, 2000; Moreno and Ubeda, 2006), I assume that the costs of pre-building capacity are sunk in the short term. This implies that capacity is either constant or accumulates along the path of play and relates the present study to "games of accumulation" (Romano and Yildirim, 2005).

equilibrium, and that Stackelberg leadership can be sustained without asynchronous timing and without retaliations against deviations of followers (i.e. in stationary equilibria of repeated games).[2]

If sales timing dominates, then the results on forward-trading derived by Allaz and Vila (1993) apply. Allaz and Vila (1993) consider T-stage games where the firms may sell forward (some of) their eventual output in stages $t<T$ and they set production quantities in stage $t=T$. Contrary to the implications of production timing, the possibility of sales timing does not affect the dimensionality of the equilibrium set. The equilibrium outcome is unique, but competition is intensified in relation to Cournot and the outcome actually converges to Bertrand as T tends to infinity.[3] Mahenc and Salanié (2004) show that forward trading has the opposite effect–to weaken competition–if firms compete in prices.

My analysis shows that neither production timing nor sales timing dominates the other in the generalized model. Rather, the equilibrium structure merges results from both streams of literature. The outcome set is a continuum that extends the Cournot–Stackelberg endogeneity to additionally include the Allaz–Vila outcome, and in case the goods are homogenous, the Allaz–Vila outcome converges to the Bertrand outcome as T approaches infinity. I derive the outcome set for oligopolies producing homogenous goods, which also shows how the Cournot–Stackelberg endogeneity generalizes to oligopoly, and for duopolies producing heterogenous goods, which additionally provides the Allaz–Vila prices for heterogenous goods.

These results highlight that the mode of competition may be entirely endogenous in oligopolistic industries. The various equilibrium outcomes may result in ex-ante equivalent industries if sales timing and production timing interact. Thus, if firms anticipate Cournot, then they are best off playing according to Cournot, if firms anticipate Stackelberg (with an arbitrary leader–follower assignment), then Stackelberg results, and so on. The firms' anticipations, in turn, may be given by historical precedents or social norms. The set of equilibrium outcomes will be characterized using a novel indexation of oligopoly equilibria that is derived from the first order conditions. The indexation links the classic modes of competition–Cournot, Bertrand, Stackelberg, and Allaz–Vila–in terms of conjectural variations and the equilibrium analysis rationalizes the corresponding conjectures. Finally, the analysis shows that forward trading of quantity setting firms is socially efficient in general (i.e. also in case the goods are complements).

Section 2 defines the notation. Section 3 derives requisite preliminary results, introduces the equilibrium index, and extends the basic Cournot–Stackelberg endogeneity to oligopoly. Section 4 analyzes the general model of oligopolists producing homogenous goods, and Section 5 concerns the case of duopolists producing heterogenous goods. Section 6 concludes. The proofs are relegated to the appendix.

2. The base model

Initially, the focus is on two-stage oligopoly games in markets for homogenous goods. Further notation will be introduced when the base model is augmented. Firms are denoted as $i \in N=\{1, \ldots, n\}$, where $n<\infty$. In stage 1, the *planning phase*, each firm $i \in N$ chooses “capacities” z_i (e.g. by pre-ordering input factors) and concludes forward contracts for y_i units of output (e.g. with retailers). In stage 2, the *production phase*, they choose the quantities x_i to be produced. The players act simultaneously in each stage, and the choices made in stage 1 are common knowledge in stage 2. The unit costs of pre-building capacity are $\gamma_i \in [0, a)$. In case a quantity $x_i>z_i$ is chosen in stage 2, the pre-built capacity is extended at unit costs $c_i \geq \gamma_i$. There are no costs of production besides the costs of capacity. The inverse demand function is $p(\mathbf{x})=a-b\sum_{i\in N}x_i$, with a and $b>0$. The forward sales are priced competitively in that the eventually resulting market price is anticipated correctly in equilibrium.

Throughout this paper scalar values and functions are set in italics, e.g. capacities z_i, vectors are set in boldface type, e.g. $\mathbf{z}=(z_i)_{i\in N}$, sets of scalars are denoted by capital letters, e.g. $Z_i \ni z_i$, and sets of vectors are denoted by capital letters set in boldface type, e.g. $\mathbf{Z}=\times_{i\in N}Z_i$.

Definition 2.1. Base game

The base game has two stages. For all $i \in N$, the strategy is a triple (z_i, y_i, x_i) where $z_i \in Z_i \subseteq \mathbb{R}_+$, $y_i \in Y_i \subseteq \mathbb{R}_+$, and $x_i: \mathbf{Z}\times\mathbf{Y}\to X_i$ with $X_i \subseteq \mathbb{R}_+$. Strategy profiles are denoted as $(\mathbf{z}, \mathbf{y}, \mathbf{x})=(z_i, y_i, x_i)_{i\in N}$. In stage 2 (when the forward trade price $\overline{p}^f$ is fixed), the profit function is, for all $i \in N$,

$$\Pi_i^S(\mathbf{x}|\mathbf{z},\mathbf{y}) = (x_i-y_i)\cdot\left(a-b\sum\nolimits_j x_j\right)+\overline{p}^f\cdot y_i-c_i\cdot max\{x_i-z_i,0\}-\gamma_i z_i. \quad (1)$$

In stage 1, the profit function is, for all $i \in N$,

$$\Pi_i^L(\mathbf{z},\mathbf{y},\mathbf{x}) = x_i(\mathbf{z},\mathbf{y})*p^f(\mathbf{z},\mathbf{y},\mathbf{x})-c_i\cdot max\{x_i(\mathbf{z},\mathbf{y})-z_i,0\}-\gamma_i z_i. \quad (2)$$

where the market price for forward trades equates with the anticipated market price conditional on $(\mathbf{z}, \mathbf{y})$, i.e.

$$p^f(\mathbf{z},\mathbf{y},\mathbf{x}) = a-b\cdot\sum_{i\in N}x_i(\mathbf{z},\mathbf{y}). \quad (3)$$

The no-arbitrage condition in Eq. (3) follows the existing literature, e.g. Allaz and Vila (1993). In the following analysis, I focus on subgame-perfect equilibria (SPEs) in pure strategies.

3. Preliminary analysis and benchmark results

3.1. Outcome uniqueness in production phase

In stage 2 of the base game, the production phase, the firms choose quantities (x_i) contingent on their capacity pre-builds (z_i) and forward sales (y_i). In standard Cournot models with linear demands and costs, the quantities chosen in equilibrium are unique. In stage 2, quantity competition and linearity apply too, but the discontinuity at the capacity limit implies that uniqueness in the production phase is less obvious than in standard models. Establishing outcome uniqueness in the production phase is important, however, to understand that the indeterminacy of the mode of competition originates in the planning phase (as one would expect) rather than the production phase.

The first result establishes outcome uniqueness in stage 2 and additionally characterizes the equilibrium outcome. To gain intuition, define the indicator $I_{x_i>z_i}$, i.e. it evaluates to 1 if $x_i>z_i$, and consider the marginal profit of i. This derivative is well defined for all $x_i \neq z_i$.

$$\frac{\partial\Pi_i^S}{\partial x_i} = p-b(x_i-y_i)-c_i\cdot I_{x_i>z_i}. \quad (4)$$

The marginal profit is piecewise linear in x_i and discontinuous at $x_i=z_i$. The non-standard characteristic of capacity pre-builds is that, if

[2] Another branch of literature, including e.g. Hamilton and Slutsky (1990), Robson (1990), and van Damme and Hurkens (1999), studies endogenous timing in duopoly. As Matsumura (1999) shows, endogenous Stackelberg does typically not result if there are more than two firms, and in general, models of endogenous timing are restrictive in the sense that firms can produce only in one of two or more initially feasible periods. Romano and Yildirim (2005) discuss this in more detail.

[3] Independently, Bolle (1993) and Powell (1993) reached similar conclusions for $T=2$. Subsequently, Ferreira (2003) derives a Folk theorem in case there is no final trading period, Liski and Montero (2006) show that forward trades simplify penal strategies in repeated oligopoly, Su (2007) provides a general existence result, Allaz (1992) and Hughes and Kao (1997) discuss demand uncertainty, and Le Coq and Orzen (2006) and Brandts et al. (2008) investigate the model experimentally.