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Dynamic feedback Stackelberg games with alternating leaders

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Abstract

In all past researches on dynamic Stackelberg games, the leader(s) and the followers are always assumed to be fixed. In practice, the roles of the players in a game may change from time to time. Some player in contract bridge, for example, acts as a leader at some stage but as a follower at the subsequent stage, which motivates the Stackelberg games with unfixed leaders. We aim to analyze the dynamic Stackelberg games with two players under such circumstances and call them dynamic Stackelberg games with alternating leaders. There are two goals in this paper. One goal is to establish models for a new type of games, dynamic Stackelberg games of alternating leaders with two players. The other goal is to extend dynamic programming algorithms to discrete time dynamic Stackelberg games with alternating leaders under feedback information structure.

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Keywords: Game theory; Discrete time dynamic models; Stackelberg games; Dynamic programming; Alternating leaders

1. Introduction

Stackelberg (leader–follower) games have a wide variety of applications. In a Stackelberg game, one player acts as a leader and the rest as followers. The problem is then to find an optimal strategy for the leader, assuming that the followers react in such a rational way that they optimize their objective functions given the leader's actions. This is the static bilevel optimization model introduced by von Stackelberg [14]. There exists extensive research on bilevel optimization [1,4,12]. However, the studies on dynamic bilevel optimization are relatively scarce. When players interact by playing a similar stage game numerous times, the game is called a dynamic, or repeated game. Unlike static games, players have at least some information about the strategies chosen on others and thus may contingent their play on past moves. Dynamic bilevel optimization was first considered by Chen and Cruz [3], Simaan and Cruz [13], and subsequently studied by a number of authors [2,8,10,9,6,5].

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The discrete time dynamic optimization problems have many applications in economics and management sciences, see an excellent monograph [2] on dynamic games. In the traditional dynamic Stackelberg games, the positions of leader(s) and followers are assumed to be fixed throughout the game. In many practical games, the leader may change at each stage. In this paper, a new type of dynamic Stackelberg game is brought forward, in which the leaders are unfixed. Moreover, dynamic programming approach is extended to the new problems with feedback information. The following examples are presented to motivate the new model.

Example 1 (*Tolls on the transportation network*). We consider the revenues raised from tolls set on a transportation network. Assume there are two ways, A and B, between two places, and A is highway. When traffic is seriously jammed by the way B, the great majority of drivers is willing to pass the corresponding acres of A to save time if the tolls are not too high. At this stage, if the tolls are set too high, traffic will be effected negatively. On the other hand, low toll values will also yield low revenues [7]. Thus, at this situation, one strikes the right balance by maximizing total revenues, subjecting to the network users. This induces a two-level problem and the toll station plays the leading role at this stage.

When the road B is not crowded, the drivers can spend a little more time without the highway A. The drivers, at this situation, strike to balance the saved time, subjecting to the toll of the corresponding arcs. This also yields a two-level problem and the drivers now play the leading roles in decision making at this stage.

Example 2 (*Stock corporation*). A stock corporation, also referred to as the "general corporation" or "open corporation". A general corporation is allowed a broad spectrum of flexibility, thanks to the general corporation laws of Delaware and the legal cases that have set a 200 year consistent pattern of respecting good-faith management decisions.

The stockholders are the owners of the company. Typically, holders of common stock have the right to one vote for each share they own to elect the members of the Board of Directors and to vote on certain other matters of major significance to the company. (According to the Delaware General Corporation Law every corporation must have one class of Common Stock. The "rules" about Common Stock are prescribed by law: each share of stock carries one vote, and common shareholders are entitled to their prorata share of dividends.)

Any stockholder who holds a majority of the shares of issued stock can control the company. This is sometimes referred to as a "majority shareholder". Majority shareholders take on a heightened responsibility to minority shareholders.

Actually, the shareholder with a majority of the shares plays the leading role in a stock corporation. It is consequently a Stackelberg game at each stage. In the long run, the shareholder with a majority of the shares is changed according to the number of shares. It is therefore inconsistent with the traditional dynamic Stackelberg games. In this dynamic game, the leader is determined by the current state and is not fixed.

Example 3 (*Contract bridge*). Bridge is played with a standard 52-card deck, or pack. Two decks are customarily used for convenience, although you can get along with just one. The deck is divided into four suits which, like military personnel, have specific rank and insignia: Spades ♠, highest. Hearts ♡, second-highest. Diamonds ◇, third-highest. Clubs ♣, lowest.

Each suit contains 13 cards: ace (highest), king, queen, jack, ten, nine, eight, seven, six, five, four, three, and two or "deuce" (lowest). These cards are often abbreviated (in order): A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, 2. The five most powerful cards in each suit (ace through ten) are accorded the privileged title of honor cards; the lower cards (nine through deuce) are referred to as spot cards. The rank of the cards within a suit applies to the phase of bridge called the play. In order to cram a great deal of information into a small amount of space, all bridge writers use diagrams and within them refer to cards by means of symbols. Thus, in a bridge diagram the ace of spades is denoted by ♠ A, the seven of diamonds by ◇ 7, the jack of clubs by ♣ J, and so on. If one player held the ace, king, ten, and seven of spades, this would be expressed concisely as ♠ A K 10 7.

Bridge is a game for four players. Unlike some activities in which everyone is out for himself or herself, bridge is a partnership game. Two of the contestants sit opposite each other and are partners; the other two participants, who also sit facing each other, are also partners. Thus, each player has an opponent on either side and a partner across the table. In bridge literature, the players are often referred to by compass directions, so North and South are partners and play against East and West, who are also partners.

In every game, each player will have 13 cards (called a hand). Each deal is divided into two major phases: bidding and play. During the bidding, which takes place first, the number of tricks that each side must win in order to capture the laurels of victory is determined. Then, the play ensues and each side tries to fulfill its commitment. However, even