

Valuation and martingale properties of shadow prices: An exposition

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Abstract

Concepts of asset valuation based on the martingale properties of shadow (or marginal utility) prices in continuous-time, infinite-horizon stochastic models of optimal saving and portfolio choice are reviewed and compared with their antecedents in static or deterministic economic theory. Applications of shadow pricing to valuation are described, including a new derivation of the Black–Scholes formula and a generalised net present value formula for valuing an indivisible project yielding a random income. Some new results are presented concerning (i) the characterisation of an optimum in a model of saving with an exogenous random income and (ii) the use of random time transforms to replace local by true martingales in the martingale and transversality conditions for optimal saving and portfolio choice. © 2000 Published by Elsevier Science B.V. All rights reserved.

JEL classification: D81; D9; C61; D46

MSC: 93E20; 90A09; 90A16; 60G44; 49K45

Keywords: Valuation; Investment; Optimisation; Continuous time; Martingales; Transversality; Time change

0. Introduction

It is well known that an optimal plan for saving and portfolio choice, over an infinite horizon in continuous time, can be characterised by means of the martingale and transversality properties of the associated shadow prices — the shadow price, or ‘marginal utility price’, of an asset or portfolio being defined as the product ($y = zv$) of its returns or market price process $z = z(\omega, t)$ and the

marginal utility process $v = v(\omega, t)$ calculated along an optimal plan. The present essay expounds and extends these properties, with special reference to their place in the economic theory of value and their applications to security and project valuation.

Regarding the theory of value, the essential points to be made at the outset are that the martingale properties of shadow prices may be regarded as generalisations to a dynamic stochastic setting of the principle of equi-marginal utility, while the transversality conditions correspond to the principles that an (unsatiated) consumer should spend the entire budget and that redundant resources attract zero prices. This foundation in fundamental concepts of economic theory lends a treatment of valuation based on marginal utility prices a remarkable degree of universality and transparency, and often affords insights which lead to simple and intuitively acceptable formulas.

The literature on valuation is fragmented, often proposing different theories and techniques for different types of assets and different situations. Thus there are theories for the valuation of ‘underlying’ securities, for derivatives and for indivisible projects, theories for complete and for incomplete markets, for continuous and for discrete time, for finite and for infinite horizons, and so forth. In general texts, the various models tend to be presented seriatim, perhaps in ascending order of mathematical difficulty or generality, with a profusion of mathematical methods and assumptions, often chosen for technical convenience in a particular discussion. Inevitably, the models are largely inspired by ideas from economic theory, but contact with these ideas (particularly in their older and simpler forms) is easily lost in the mass of technicalities. A substantial part of the recent literature restricts attention to the valuation of derivative securities in the setting of a complete market, taking as given the price processes of the ‘underlying’ securities. Interest centres on special mathematical methods which are convenient for this class of problems and contact with the general theory of value is minimal from the outset — the only major principles borrowed from it being (loosely speaking) that more money is better than less, and one or more forms of the ‘law of one price’.¹

The methodological stance of the present essay, briefly stated, is that the natural way to develop the subject of financial asset valuation is to start with the general concepts and principles of the economic theory of value, to classify particular problems and techniques according to criteria suggested by this

¹ Some practitioners even affect to despise more general theories of economics based on concepts of utility etc, and regard financial valuation as a ‘stand-alone’ branch of applied mathematics which finds its practical expression in so-called ‘financial engineering’. Enthusiasm for methods of hedging and valuation of derivatives in complete markets, and for associated methods of computation, seems often to obscure the fact that these techniques do not provide a general theory of valuation and that they are liable to give at best only imprecise results when applied beyond their proper domain.

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