

# Valuation of European Currency Options in Financial Engineering

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## Abstract

In this paper, we try to solve the valuation of currency option in financial engineering. We use a generalized jump-diffusion system to describe the spot Foreign Exchange (FX) rate and apply regime switching model to describe the domestic and foreign risk-free interest rate and the appreciation rate and the volatility. And the regime switching model is based on a continuous time finite state Markov process. Under the minimal martingale measure, we obtain a system of partial-differential-integral-equations satisfied by the European currency option prices. Our model provides the flexibility to model different kinds of dynamics in FX rate. At last, we present a simulation of option pricing with the special case of compound Poisson jump and we can find the effects of the parameters on the prices.

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*Keywords:* Jump-diffusion system; Minimal martingale measure; Currency Option pricing; Regime switching; Financial Engineering

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## 1. Introduction

Currency option is an important derivative tool in foreign exchange market and its pricing and hedging have also been a fundamental subject in financial engineering. After the seminal work of Black and Scholes [1] and Merton [2] on European stock option pricing, there were [3] and [4] as the pioneering works in this area. In their models, they applied the Geometric Brownian Motion with constant appreciation rate and the volatility and then valued the option under risk-neutral measure.

However, many following empirical studies have defied the assumption of Geometric Brownian Motion of dynamics of the spot exchange rate and they ask for more realistic models. As a result, a lot of papers are devoted to the modification of the basic Geometric Brownian Motion. Firstly, in reality the parameters of the spot FX dynamics may depend on the state of the economy. In this case, many studies have introduced the Markov Regime Switching Model denoted by a finite state Markov chain to capture the realities. The switching of economy states can be caused by the structural breaks of the economy situations and business cycles. The early work on the introduction of regime switching model includes Quandt [5] and Goldfeld and Quandt [6]. Ever since Hamilton [7] introduced the regime switching time series to economics, the regime-switching models have obtained great development in financial research, even in actuarial science. Secondly, some rare events (major political changes, a natural disaster in a major economy, or the release of unexpected economic data) may result in the brusque variations in spot FX rate. Since the variations cannot be described by the usual diffusion models, we need to consider the jump diffusion model to capture the jump in rates. [8] applied the jump diffusion model for spot FX rate.

The improvement of the ability of the model to explain the reality is accompanied with the difficulty to evaluate

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the derivatives. An important property of regime switching model or jump diffusion model or their combination is the incompleteness of the financial market. It means that the contingent claim cannot be hedged perfectly. Harrison and Kreps [9], Harrison and Pliska [10, 11] found the relationship between the absence of arbitrage opportunity and the existence of the equivalent martingale measure. They showed that there would be infinitely many equivalent martingale measures and the problem was how to select a consistent measure from that set of measures. Föllmer and Sondermann [12], Föllmer and Schweizer [13], Schweizer [14] and Davis [15] provided different methods to choose a pricing measure on basis of different optimization criteria. In this paper, we follow risk minimizing method proposed by Föllmer and Schweizer [13]. In an incomplete market, for non-attainable claims, it's desirable to find the strategies with the minimization of future risk exposure at any time in the future. Föllmer and Sondermann [12] first introduced the risk minimizing hedging to address the pricing of the options in an incomplete market and this was further studied by Föllmer and Schweizer [13] in which they provided the Föllmer and Schweizer decomposition of the discounted asset price.

In this paper, we consider the pricing of the European currency options when the dynamics of spot FX rate are driven by a generalized Markov-modulated jump-diffusion model using the risk minimizing methods to choose the equivalent martingale measure. Our work is different from the previous existing studies. [16] studied the currency option pricing under two-factor Markov-modulated stochastic volatility model. It used the Esscher Transform to choose the equivalent martingale measure which was different from our minimal martingale measure. And there was no jump part in their model. [17] presented the formula of European currency option price when the spot foreign exchange rate followed the fractional Brownian motion with jumps. [18] studied the European currency option valuation with spot foreign exchange rate described by a diffusion model with compound Poisson jumps and regime switching. They also used the Esscher Transform to select the equivalent martingale measure. Furthermore, since we consider the option pricing under the generalized form of the dynamics of spot FX rate, our work will provide the flexibility to the market participants to choose the special models they need to describe spot FX rate in markets. And under the minimal martingale measure, we derive the system of coupled partial differential integral equations satisfied by the option prices. At last, we do the numerical experiment of option prices when we choose the compound Poisson process as the concrete jump part in our original model and we consider the qualitative behavior of the price with different parameters.

This paper is structured as follows. Section 2 presents the general formula of spot FX rate and the derivation of minimal martingale measure. Section 3 gives the system of coupled partial differential integral equations. In Section 4, we discuss the simulation results conditioned on the concrete jump process. At last, Section 5 concludes the whole paper.

## 2. The Model

In this section we introduce two basic elements of the model: the spot FX rate and the minimal martingale measure.

We consider only two currencies in FX market, called the domestic currency and the foreign currency. Let  $S_t$  denote the spot FX rate process, representing the domestic price of one unit of foreign currency at time  $t$ . Then we let the dynamic of  $S_t$  follow the generalized jump-diffusion regime switching model. We also assume that the instantaneous market interest rates in domestic and foreign markets are controlled by a continuous-time finite-state Markov chain. And then we can introduce the dynamics of the spot FX rate.

Fix a complete probability space  $(\Omega, F, P)$ , where  $P$  is a real-world probability.  $T < \infty$  denotes a fixed, finite time horizon. Let  $X := \{X_t\}_{0 \leq t \leq T}$  be a continuous-time Markov chain on  $(\Omega, F, P)$  taking values in a finite state space  $\mathcal{X} := \{1, 2, \dots, n\}$ . We interpret the state of  $X$  as the state of the economy. We suppose that the risk-free interest rate, appreciation rate and volatility of the risky asset depend on the state of the economy. The Markov chain  $X_t$  is governed by

$$P(X_{t+\Delta} = j | X_t = i) = \lambda_{ij}\Delta + o(\Delta) \quad i \neq j, \tag{1}$$

where  $\lambda_{ij} \geq 0, i \neq j$ ; and  $\lambda_{ii} = - \sum_{j=1}^n \lambda_{ij}$ . Let  $\Lambda = \{\lambda_{ij}\}$  denote the generating  $Q$ -matrix of the chain.

It would be convenient to write (1) in an equivalent way where  $\{X_t\}$  is represented as a stochastic integral with

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