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Optimal decisions for the loss-averse newsvendor problem under CVaR

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ABSTRACT

Most of the existing literature about the newsvendor problem mainly focused on choosing an optimal order quantity to maximize the expected profit of a risk-neutral newsvendor. However, some studies pointed out that in practice managers' decisions as to order quantities always deviate from the profit-maximization order quantity, which is referred to as decision bias in the newsvendor problem. Then, many studies introduced other preferences rather than risk neutrality of the newsvendor to explain such a decision bias. This paper thus introduces loss aversion to the research of the newsvendor decision bias. We propose a new definition to the loss-averse newsvendor problem–legacy loss, which is defined either as the loss for excess order or the shortage penalty for lost sales when sales time is due. The optimal decisions of a loss-averse newsvendor are obtained through the following three ways: (i) minimizing the expected legacy loss, (ii) minimizing CVaR of legacy loss by combining the CVaR measure in risk management, (iii) minimizing the combination of expected legacy loss and CVaR of legacy loss; and are compared with other existing results. Besides, we present some properties of the optimal decisions as well as relations among them. Some numerical examples are given to illustrate the obtained results.

1. Introduction

Newsvendor problem is well-known in inventory control literature, and has been widely applied to settings such as production planning and supply chain management (Petruzzi and Dada, 1999; Khouja, 1999; Dai and Meng, 2015; Wu et al., 2015). In the classical newsvendor problem (which assumes that the newsvendor is risk-neutral), where perishable products with stochastic demand are sold in a short selling season, a newsvendor needs to decide before the selling season an optimal order quantity of the product to maximize his/her expected profit. If the order quantity is greater than the realized demand of the market, the newsvendor then has to dispose the excess order as a loss. Otherwise, if the order quantity is lower than the realized demand of the market, there is some shortage penalty for the lost sales to the newsvendor. Therefore, to maximize his/her expected profit, the newsvendor must find a balance between ordering too much and too little. In other words, the studies of the classical newsvendor problem always focus on choosing an optimal order quantity to maximize the expected profit $E[P(q)]$ (E is the expectation operator) of a risk-neutral newsvendor, with the profit function $P(q)$ given as

$$P(q) = r\min\{q,d\} - cq + s(q-d)^+ - p(d-q)^+,$$

where $X^+ = \max\{0,X\}$, r is the retail price of unit product, q is the order quantity of the newsvendor, d is the realized demand of the market, c is the wholesale price of unit product, s is the salvage price of unit product and p is the shortage penalty price of unit product (in some literature, it is supposed that $p = r - c$). Here, the first item in the right hand of the above equality represents the sales income of the newsvendor, the second the order cost, the third the salvage income of excess order and the last the shortage penalty for lost sales. However, some recent studies found that, in practice, the realized order quantities of the managers are not always consistent with the above expected profit-maximizing order quantities. In other words, in practice managers' choices always deviate from those that maximize their expected profit. For example, Schweitzer and Cachon (2000) conducted two experiments in which the decisions of a newsvendor across different profit conditions were investigated and obtained the following conclusion: orders for high-profit products are lower than the expected profit-maximizing order quantity, while orders for low-profit products are higher than the expected profit-maximizing order quantity. This deviation of the realized order quantity of the manager in practice from the expected profit-maximizing

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order quantity is referred to as "decision bias" in the newsvendor problem (Schweitzer and Cachon, 2000; Wang and Webster, 2009; Chen et al., 2014). Then, many researchers seek to find possible reasons to explain decision bias in newsvendor problem and some of them introduce preferences other than risk neutrality (e.g. risk-aversion and loss-aversion) to explain the newsvendor decision bias and get some interesting results.

In recent years, some unpredictable disasters (e.g. earthquakes and economic crises) have disrupted supply operations and brought great losses to a newsvendor. This teaches the newsvendor to be more loss-averse in selecting his/her order quantity. In fact, for a loss-averse manager who is averse to losses as compared to obtaining the same size of profit, he/she pays more attention to loss reduction rather than profit maximization. Such loss-averse managers are even willing to trade off lower expected profit with downside protection against possible loss. In other words, a loss-averse manager concerns about the maximization of the profit, but more about the minimization of the losses. Wang and Webster (2009) introduced loss aversion to describe newsvendor's decision bias and showed that if shortage penalty for lost sales is not negligible, a loss-averse newsvendor may order more or less than a risk-neutral newsvendor, which verifies the existence of decision bias in newsvendor problem. Moreover, it is also proved that a loss-averse newsvendor's optimal order quantity may increase in the wholesale price and decrease in the retail price, which can never occur in the risk-neutral newsvendor problem. In fact, loss aversion is both intuitively attractive and well supported in fields like finance and economics. For example, the empirical studies by Shapira (1986), based on interviews with 50 American and Israeli executives, and by MacCrimmon and Wehrung (1996), based on questionnaires from 509 high-level executives of American and Canadian firms, showed managers' decision-making behavior in the real world is always consistent with that of loss aversion. Moreover, a stream of literature closely related to the successful applications of loss aversion can be found in areas such as financial markets (Benartzi and Thaler, 1995), marketing (Putler, 1992), organizational behavior (Fiegenbaum and Thomas, 1988) and labor supply (Camerer et al., 1997). Evidently, loss aversion provides an alternative choice to explain decision bias in newsvendor problem. However, to our best knowledge, there is little study that addresses loss-aversion in the newsvendor problem.

As mentioned above, compared with maximizing the profit, a loss-averse newsvendor prefers to choose an order quantity to minimize his/her loss. Especially when the sales time is due, it is necessary and meaningful for a loss-averse newsvendor to minimize his/her losses in that time, that is

$$L(q)=(c-s)(q-d)^{+}+p(d-q)^{+}. \tag{1}$$

Here, the first item in the right hand of the above equality represents the loss for excess order, while the second the shortage penalty for lost sales. We define this kind of loss $L(q)$ when the sales time is due as "legacy loss" in newsvendor problem. Then we have the following relation between the profit $P(q)$ and legacy loss $L(q)$ about an order quantity q:

$$P(q)=(r-c)\min\{q,d\}-L(q).$$

Here, the first item in the right hand of the above equality represents the sales profit of the newsvendor during the selling season. It is easily seen from the above equality that, no matter what sales profit the newsvendor gets during the selling season, if the legacy loss is great, then the profit of the newsvendor will be less. Moreover, the profit $P(q)$ of the newsvendor can be re-written as

$$P(q)=\begin{cases} rq-cq-p(d-q) & q<d,\\ rq-cq & q=d,\\ rd-cq+p(q-d) & q>d. \end{cases}$$

By (1), the legacy loss $L(q)$ of a newsvendor attains its minimum when $q=d$. This is equal to the fact that $P(q)$ attains its maximum when $q=d$ (which is easily proved with the above expression of $P(q)$). Further, it is obvious that what a newsvendor wants to do is to choose an order quantity which minimizes the gap between this order quantity and the realized demand of the market. If we choose an order quantity which minimizes the legacy loss $L(q)$, then it follows from (1) that the excess order $(q-d)^{+}$ and the lost sales $(d-q)^{+}$ both attain their minimums, then the gap between the order quantity and the realized demand is minimized. Thus, minimizing the legacy loss of a loss-averse newsvendor provides a more practical view of point for the loss-averse newsvendor problem.

Therefore, this paper focuses on how to decide order quantities for the loss-averse newsvendor to minimize the above legacy loss $L(q)$. First, we obtain an optimal order quantity that minimizes the expected legacy loss $E[L(q)]$ of a loss-averse newsvendor who is insensitive to loss variations. It is proved that this optimal order quantity in minimizing the expected legacy loss is smaller than that in maximizing the expected profit. Moreover, it is easily checked that this optimal order quantity is increasing in the shortage penalty price p and the salvage price s, and decreasing in the wholesale price c respectively. However, the significance of the expected performance measure highly depends on its associated variance. If the variance of the legacy loss is large, the chance of deviating from the expected value of legacy loss will be high. In other words, this expected performance measure ignores the risks arising from the fluctuation of market demand, which may lead to great losses to the loss-averse newsvendor. Then in order to measure and control such risks, we introduce the CVaR measure for the loss-averse newsvendor problem. The CVaR is known as a risk measure which is coherent and consistent with the second (or higher) order stochastic dominance. In particular, the consistency with the stochastic dominance implies that minimizing the CVaR never conflicts with minimizing the expectation of any risk-averse loss function. Moreover, the lower partiality of the CVaR plays an important role in preserving convexity of the loss. Here, for a loss-averse newsvendor who wants to control the potential risk, we introduce the following objective by adopting the CVaR measure:

$$\min_{q\geq 0}\ \mathrm{CVaR}_{\alpha}[L(q)]=E[L(q)|L(q)\geq \mathrm{VaR}_{\alpha}(L(q))].$$

The optimal order quantity in minimizing this objective minimizes the conditional expected value of legacy loss which exceeds a certain quantile level $\mathrm{VaR}_{\alpha}(L(q))$. In other words, this optimal order quantity implies that the expected legacy loss is minimum provided it is above the quantile level $\mathrm{VaR}_{\alpha}(L(q))$ and thus guarantees the minimum expected legacy loss at the confidence level α for a loss-averse newsvendor. It is proved that the optimal order quantity in minimizing CVaR of legacy loss may increase or decrease at the confidence level α, and hence be bigger or smaller than the expected-legacy-loss-minimizing order quantity above. Moreover, it is proved that this optimal order quantity increases in the shortage penalty price p and the salvage price s, and decreases in the wholesale price c, which is