Int. J. Production Economics 135 (2012) 162–169

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

# Optimal pricing and order quantity for the newsvendor problem with free shipping

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ARTICLE INFO

*Article history:*
Received 30 November 2009
Accepted 19 November 2010
Available online 24 November 2010

*Keywords:*
Free shipping
Newsvendor problem
Pricing
Transportation
Quantity discount

ABSTRACT

To attract and keep customers, companies, especially those in e-business, are increasingly offering free shipping to buyers whose order sizes exceed the free shipping quantity. In this paper, given the supplier offers free shipping and the retailer faces stochastic demand, we determine the retailer's (i.e., the newsvendor's) optimal order quantity and the optimal selling price simultaneously. We consider two different ways in which price affects the demand distribution, namely price only affects the location or scale of the demand distribution. We explicitly incorporate the supplier's quantity discount and transportation cost into the models. The transportation cost function is very general, which includes those most commonly used in the literature. We numerically examine the impacts of free shipping, quantity discount, transportation cost, and demand variance on the retailer's optimal order quantity and pricing decisions. We find that even though the retailer faces uncertain demand, free shipping can effectively encourage the retailer to order more of the good and can benefit the supplier, the retailer, and the end customers. An increase in transportation cost or a decrease in purchase price will induce the retailer to order more of the good and decrease the retail price. With increasing demand variance, the retailer should order more of the good. We also find that the newsvendor can cope with demand variance by taking advantage of free shipping.

## 1. Introduction

To attract and keep customers or retailers, and encourage them to order more goods, companies, especially business-to-customer (B2C) and business-to-business (B2B) firms, are increasingly offering free shipping to buyers whose order sizes exceed the free shipping quantity (FSQ). For example, Amazon.com offers free shipping for orders over US$25; transcendusa.com,[1] an online store that sells memory flash cards, hard disks, and MP3 players, offers free shipping for orders of US$150 or more; the B2B companies 1800contacts.com,[2] a lenses supplier, and re-inks.com,[3] a printer ink cartridges supplier, provide free shipping for web orders over US$50 and US$45, respectively. A proper free shipping policy can influence customers' purchasing behaviours such as inducing buyers to place larger orders less frequently, which allows the supplier to achieve economies of scale in terms of production and distribution (Zhou et al., 2009). There is even evidence that free shipping has a greater impact on buyers than price discount. For example, a survey found that a free shipping offer that saved US$6.99 was more appealing to many buyers than a discount that cut the purchase price by US$10, and shipping and handling costs triggered 52% of the abandonment of online shopping carts (Knowledge@Wharton, 2008).

However, the literature on free shipping is very sparse. Lewis et al. (2006) empirically find that free shipping with or without conditions is very effective in generating additional sales. Yang et al. (2005) consider a free shipping problem in the B2C context. They examine the optimal shopping policy of a shopper and the retailer's endogenous choices of price and free shipping. Leng and Parlar (2005) examine a free shipping problem in the B2B context using Stackelberg game, in which the seller as the leader first sets the FSQ, then the retailer as the follower decides the purchase quantity. Their shipping cost is assumed as a continuous and smooth function of the purchase value. However, in practice, shipping cost may not be smooth and may even be discontinuous in the purchase value (Abad and Aggarwal, 2005; Ertogral et al., 2007; Ertogral, 2008; Russell and Krajewski, 1991; Swenseth and Godfrey, 2002). Zhou et al. (2009) examine the problem of a stochastic inventory system with the free shipping option. They present the optimal order policy for the single-period inventory system and a heuristic policy for the multi-period case. Yang et al. (2005) and Zhou et al. (2009) assume that the retailer charges a fixed fee $K$ for shipping an order that is below the FSQ, i.e., $K$ is independent

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[1] http://ec.transcendusa.com/term.asp?TID=14

[2] http://www.1800contacts.com/

[3] http://www.re-inks.com/

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doi:10.1016/j.ijpe.2010.11.019

of the order value. In B2C transactions, it is reasonable to assume that the shipping cost for any order that is below the FSQ is a fixed fee since a customer's order quantity is usually relative small. However, this assumption may not be valid for a supplier–retailer system or B2B transactions because the order quantities are usually very large and the quantity dispersion of the orders is usually large.

In practice, suppliers and shippers commonly offer quantity discounts and freight discounts to retailers. However, all the above literature does not capture these discount schedules (Leng and Parlar, 2005 implies freight discount, but their shipping cost is a continuous and smooth function of the purchase value). Leng and Parlar (2005) and Zhou et al. (2009) do not incorporate pricing decisions into their models. In fact, the free shipping policies of suppliers have a significant impact on the optimal pricing decisions of retailers. To fill this gap, Hua et al. (under review) investigate the optimal order lot sizing and pricing strategies with free shipping based on the EOQ model in the B2B context. They incorporate quantity discount, freight discount, and pricing decision into the model. Their transportation cost function is very general, which includes all the transportation cost functions used in the above studies, except Leng and Parlar (2005), and other most commonly used ones in the literature as special cases. However, they assume that the demand faced by the retailer is deterministic or price-dependent.

This paper address a new free shipping problem in the B2B context where the demand faced by the retailer is stochastic and the retail price affects the demand distribution. We determine the retailer's optimal order quantity and the retail price simultaneously. In view of the fact that the product life cycle is shortening and more products possess the attributes of fashion or seasonal goods, we use the classical newsvendor problem framework to deal with the problem. We consider two different ways in which price affects the distribution of demand, namely price only affects the location or scale of the demand distribution. The newsvendor problem that incorporates the pricing decision was originally investigated by Whitin (1955) and Karlin and Carr (1962). Petruzzi and Dada (1999) and Yao et al. (2006) present reviews and extensions of the problem. Agrawal and Seshadri (2000) examine the newsvendor problem in which a risk-averse retailer faces uncertain customer demand and makes purchase order quantity and selling price decisions with the objective of maximizing the expected utility. Different from the above literature, we examine the problem with free shipping and incorporate both quantity and freight discounts into the models, which are commonly faced by the newsvendor. We present the methods to find the optimal order quantity and pricing decisions and numerically examine the impacts of free shipping, quantity discount, transportation cost, and demand variance on the newsvendor's optimal order quantity and pricing decisions. We obtain some new and interesting management insights that are relevant to both theory and practice.

This paper is organized as follows: In Section 2 we present the transportation cost function. In Sections 3 we formulate the free shipping problem when price only affects the location or scale of the demand distribution, and examine the optimal order and pricing decisions. In Section 4 we present numerical results on the impacts of the FSQ, quantity discount, transportation cost, and demand variance on the newsvendor's optimal order quantity and pricing decisions. Finally we conclude the paper and suggest topics for future research.

## 2. Quantity discount and transportation cost

In this section we introduce the quantity discount schedule and the transportation cost function for the problem under study. To encourage retailers to order more goods, suppliers usually offer quantity and freight discounts to their retailers, which benefit both suppliers and retailers by achieving economies of scale. The discount schedules commonly examined in the literature are either all-unit discount or incremental discount. All-unit discount means that purchasing larger quantities results in progressively lower unit purchase prices for the entire lot, while incremental discount means the progressively lower unit purchase prices only apply to units purchased above specified quantities. Since all-unit discount is the most common and prevalent in practice, we assume the quantity discount in this paper is of the all-unit type.

In practice, shipping service providers usually offer freight rate discounts to customers, which are similar to quantity discounts but are usually based on weight, applicable to units of a single product. In this paper we focus on a single product, so the transportation cost can be based on units. Transportation cost function with all-unit discount can be classified into one with over-declaration and another without over-declaration.

Following Hua et al. (under review), we adopt in this paper a general transportation cost function as follows:

$$FC(Q) = u_i + v_i Q,\ \ u_i \ge 0,\ \ v_i \ge 0, \quad Q \in [M_i, M_{i+1}),\ \ i = 0,1,2,\ldots,m, \tag{1}$$

$$u_i + v_i M_{i+1} \ge u_{i+1} + v_{i+1} M_{i+1}, \quad i = 0,1,2,\ldots,m-1,\ \ M_{m+1} = +\infty, \tag{2}$$

where $M_{i+1}$, $i=0,1,2,\ldots, m-1$, in (2) are price discount points. Hua et al. (under review) shows that many transportation cost functions used in the literature are special cases of our cost function. For example, when $u_i = 0,\ v_i \ne 0$, the cost function is the all-unit freight discount without over-declaration, and when $u_k = 0,\ v_k \ne 0,\ k = 0,2,4,\ldots$, and $u_l \ne 0,\ v_l = 0,\ l = 1,3,5,\ldots$, the cost function is that with over-declaration, which is the same as the cost functions in Ertogral et al. (2007). When $u_i = K$ and $v_i = 0$, the cost function is the same as those in Yang et al. (2005) and Zhou et al. (2009). In addition, the cost functions in Abad and Aggarwal (2005) and Abad (2006) are special cases of our cost function, too. Furthermore, Hua et al. (under review) show that the transportation cost for the incremental freight discount schedule is also a special case of our cost function, so our cost function is very general.

It is noted that the price-break points in the quantity discount schedule may not be the same price-break points in the freight cost schedule, and the FSQ may not be exactly one of the above price-break points. In view of this, we arrange the price-break points in quantity and freight cost discounts and the FSQ in non-decreasing order. Without loss of generality, we assume $M_0 < M_1 < M_2 < \cdots < M_m$ and the FSQ is $M_{i0}$, say. The quantity discount schedule and transportation cost can be re-formulated easily according to these price-break points (Hua et al., under review).

There are plentiful studies on purchase decisions incorporating quantity discount and shipping cost, such as Tersine and Barman (1991), Burwell et al. (1997), Russell and Krajewski (1991), Swenseth and Godfrey (2002), Abad and Aggarwal (2005, 2006), and Ertogral et al. (2007). Differing from the above literature, we incorporate free shipping, quantity discount, and transportation cost into the order quantity and pricing decisions in this study.

## 3. The models

### 3.1. Notation and assumptions

We consider the newsvendor problem that incorporates price decision with free shipping, in which the newsvendor absorbs the freight charges when its order quantity is below the FSQ. The supplier offers its retailers free shipping, quantity discount, and freight discount. Even if a third-party logistics firm is used to deliver the good to retailers, it usually offers freight discounts to his