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Multi-product newsvendor problem with value-at-risk considerations

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ABSTRACT

We consider the single period stochastic inventory (newsvendor) problem with downside risk constraints. The aim in the classical newsvendor problem is maximizing the expected profit. This formulation does not take into account the risk of earning less than a desired target profit or losing more than an acceptable level due to the randomness of demand. We utilize Value at Risk (VaR) as the risk measure in a newsvendor framework and investigate the multi-product newsvendor problem under a VaR constraint. To this end, we first derive the exact distribution function for the two-product newsvendor problem and develop an approximation method for the profit distribution of the N-product case ($N>2$). A mathematical programming approach is used to determine the solution of the newsvendor problem with a VaR constraint. This approach allows us to handle a wide range of cases including the correlated demand case that yields new results and insights. The accuracy of the approximation method and the effects of the system parameters on the solution are investigated numerically.

1. Introduction

In the vast literature of inventory control, most of the models employ maximization of the expected profit as the main optimality criterion. In recent studies, the financial risk in management of inventory systems is considered from various perspectives. There are many risk measures that are used in risk management in stochastic inventory models such as the satisficing probability maximization, utility functions, Value at Risk (VaR) and conditional VaR (CVaR).

In this study, we focus on VaR as a measure of downside risk and incorporate this risk measure in the multi-product newsvendor problem. VaR measures the maximum value of the random function or the variable in a β confidence interval, see for example, Jorion (1997), Artzner et al. (2000), and Simons (1996). VaR is a measure of the maximum potential change in the value of a portfolio of financial instruments over a pre-set horizon. VaR answers the question of how much one can lose with x% probability over a given time horizon. If a portfolio is expressed as a 95% one-day VaR of $100 million, this means that there is only a 5% chance that the portfolio will lose more than $100 million over the next day.

Multi-product newsvendor problem can also be considered as a problem of determining the best product portfolio among all the possible alternatives. Similar to the case of investing in a financial portfolio, a retailer faces a substantial risk in its ordering decisions. In a multi-product portfolio, if the retailer ends up with a high number of unsold products at the end of the season, the financial losses can be devastating. From this perspective, the objective of the retailer is maximizing the expected return while it takes calculated risks, e.g., the retailer

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knows in advance that the probability of losing more than a pre-determined level is less than the desired probability.

In this paper, we present a mathematical programming methodology to solve the multi-product newsvendor problem with a VaR constraint. The VaR constraint is expressed explicitly by using the probability distribution of the total profit.

For the two-product case, we give a compact expression that yields the total profit distribution based on the demand distributions. Once the VaR constraint can be expressed explicitly, the resulting optimization problem can be solved by a mathematical programming approach. We illustrate our approach for the cases with independent and bivariate exponential distributions.

For the multi-product case, we present an approximate method that is based on approximating the distribution of the total profit with a normal distribution following the central limit theorem. Numerical comparisons with simulation shows that this approximation is quite accurate in representing downside risk constraints.

The organization of the remaining part of this paper is as follows. In Section 2, we review the pertinent literature. We introduce the model and summarize the results for a single-product newsvendor problem with a VaR constraint in Section 3. In Section 4, the two-product newsvendor problem with a VaR constraint is formulated as a mathematical program by deriving the probability distribution of the total profit. Results for two products with independent and correlated demands are also given in this section. Section 5 extends this approach to multi-product case by approximating the probability distribution of the total profit with a normal distribution. Finally, conclusions are given in Section 6.

2. Literature review

Incorporating risk in inventory management decisions received some attention in recent years. Sankarasubramanian and Kumarasamy (1983) consider a single-period stochastic inventory problem in which it is required to determine the product order quantity which maximizes the probability of realizing a predetermined level of profit. A condition for deciding the optimal order quantity is found and explicit expressions for the optimal order quantities in three special cases are given. Schweitzer and Cachon (2000) investigate the decision bias in the newsvendor problem with a known demand distribution.

The satisfaction probability is used as an objective function in a number of studies (Lau, 1980; Lau and Lau, 1988; Li et al., 1990, 1991; Parlar and Weng, 2003). Satisficing probability is defined as the probability of exceeding a prespecified fixed target profit level. The aim is to maximize the satisfaction probability function in terms of the product order quantity. Lau (1980) solves the satisfaction probability maximization problem for a single product under the assumption of zero salvage value.

Lau and Lau (1988) consider the maximization of the probability of achieving a target profit in a two-product newsvendor problem. Solution procedures are developed to find the optimal order quantities of each product that will maximize the probability of achieving the target profit value. Li et al. (1990, 1991) present an analytical solution procedure to maximize the probability of achieving a target profit in a two-product newsvendor problem for uniformly and exponentially distributed demands respectively. Some analytical results are presented for these restrictive cases.

Parlar and Weng (2003) investigate the satisfaction probability value maximization objective in the classical newsvendor problem. They also develop a model that integrates this objective with the standard expected profit maximization objective. In our setting, the satisfaction probability function is used as a constraint of the classical newsvendor problem and the aim is to solve this model for N product case.

An alternative approach for modeling risk preferences in inventory management, has been using utility functions. Lau (1980) maximizes the expected utility of the newsvendor problem. The utility function is defined in terms of the expected value of the random profit and its standard deviation. This corresponds to the well-known mean-standard deviation trade-off approach. Eeckhoudt et al. (1995) and Bouakiz and Sobel (1992) examine the risk aversion in the newsvendor problem with an exponential utility function and show that a base-stock policy is optimal when a dynamic version of the newsvendor model is optimized with respect to an exponential utility criterion. In a recent paper, Chen et al. (2007) incorporate risk aversion through utility maximization in multiperiod inventory models involving pricing strategies.

There is a huge interest in hedging operational risks using financial instruments. Anvari (1987) uses the well-known capital asset pricing model (CAPM) in finance to investigate a newsvendor problem. Gaur and Seshadri (2005) investigate the impact of financial hedging on operational decisions in the framework of the newsvendor problem. They develop optimal hedging transactions that minimize the variance of profit and increase the expected utility for a risk-averse decision maker.

Luciano et al. (2003) investigate VaR as a risk measure in the context of a single-product multi-period Economic Order Quantity type inventory model. They present an exact analysis to obtain the VaR and also establish useful bounds. In contrast, we investigate a single-period problem but focus on the interaction of multiple products that are related through the VaR constraint.

In Tapiero (2005), an asymmetric valuation between ex-ante expected costs above and below an appropriate target cost, provides an explanation for the VaR criterion when it is used as a tool for design. This approach gives some insights regarding the selection of the VaR probability that turns out to be the ratio of the asymmetric linear cost parameters in this case. In this setting, it is proposed to optimize the planned (targeted cost) that is defined as the sum of the expected newsvendor cost and risk specification quantile (defined to be "index of risk aversion") times the standard deviation of the newsvendor cost under the assumption of normally distributed newsvendor cost function. We do not consider the design problem here and assume that a desired profit level and a corresponding level is specified exogenously. In particular,