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Quantifying economic fluctuations

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Dedicated to Professor Dr. H.A. Weidenmüller on the occasion of his 65th birthday

Abstract

This manuscript is a brief summary of a talk designed to address the question of whether two of the pillars of the field of phase transitions and critical phenomena—scale invariance and universality—can be useful in guiding research on interpreting empirical data on economic fluctuations. Using this conceptual framework as a guide, we empirically quantify the relation between trading activity—measured by the number of transactions N—and the price change $G(t)$ for a given stock, over a time interval $[t, t + \Delta t]$. We relate the time-dependent standard deviation of price changes—volatility—to two microscopic quantities: the number of transactions $N(t)$ in Δt and the variance $W^2(t)$ of the price changes for all transactions in Δt. We find that the long-ranged volatility correlations are largely due to those of N. We then argue that the tail-exponent of the distribution of N is insufficient to account for the tail-exponent of $P\{G > x\}$. Since N and W display only weak inter-dependency, our results show that the fat tails of the distribution $P\{G > x\}$ arises from W. Finally, we review recent work on quantifying collective behavior among stocks by applying the conceptual framework of random matrix theory (RMT). RMT makes predictions for "universal" properties that do not depend on the interactions between the elements comprising the system, and deviations from RMT provide clues regarding system-specific properties. We compare the statistics of the cross-correlation matrix C—whose elements C_{ij} are the correlation coefficients of price fluctuations of stock i and j—against a random matrix having the same symmetry properties. It is found that RMT methods can distinguish random and non-random parts of C. The non-random part of C which deviates from RMT results, provides information regarding genuine collective behavior among stocks. We also discuss results that are reminiscent of phase transitions in spin systems, where the divergent behavior of the response function at the critical point (zero magnetic field) leads to large fluctuations, and we discuss a curious "symmetry breaking", a feature qualitatively identical to the behavior of the probability density of the magnetization for fixed values of the inverse temperature. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In recent years, physicists have started applying concepts and methods of statistical physics to study economic problems. The word “Econophysics” is sometimes used to refer to this work. Much recent work is focused on understanding the statistical properties of financial time series. One reason for this interest is that financial markets are examples of complex interacting systems for which huge amounts of data exist and it is possible that financial time series viewed from a different perspective might yield new results. This article reviews the results of several recent studies, with emphasis on studies carried out by the authors.

(i) *The probability distribution of stock price fluctuations.* Stock price fluctuations occur in all magnitudes, in analogy to earthquakes—from tiny fluctuations to drastic events, such as market crashes. The distribution of price fluctuations decays with a power-law tail well outside the Lévy stable regime and describes fluctuations that differ by as much as 8 orders of magnitude. In addition, this distribution preserves its functional form for fluctuations on time scales that differ by 3 orders of magnitude, from 1 min up to approximately 10 days.

(ii) *Correlations in financial time series.* While price fluctuations themselves have rapidly decaying correlations, the magnitude of fluctuations measured by either the absolute value or the square of the price fluctuations has correlations that decay as a power-law and persist for several months.

(iii) *Correlations among different companies.* The third result bears on the application of random matrix theory to understand the correlations among price fluctuations of any two different stocks. From a study of the eigenvalue statistics of the cross-correlation matrix constructed from price fluctuations of the leading 1000 stocks, we find that the largest $\approx 5\%$ of the eigenvalues and the corresponding eigenvectors show systematic deviations from the predictions for a random matrix, whereas the rest of the eigenvalues conform to random matrix behavior—suggesting that these 5% of the eigenvalues contain system-specific information about correlated time evolution of different companies.

(iv) *Similarities with critical point phenomena.* We also discuss results that are reminiscent of phase transitions in spin systems, where the divergent behavior of the response function at the critical point (zero magnetic field) leads to large fluctuations. In particular, we discuss a curious “symmetry breaking” for values of Σ above a certain threshold value Σ_c; here Σ is defined to be the local first moment of the probability distribution of demand Ω—the difference between the number of shares traded in buyer-initiated and seller-initiated trades. This feature is qualitatively identical to the