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Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm and non-stochastic simulations

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ABSTRACT

This article describes the approach to computing the version of the stochastic growth model with idiosyncratic and aggregate risk that relies on collapsing the aggregate state space down to a small number of moments used to forecast future prices. One innovation relative to most of the literature is the use of a non-stochastic simulation routine.

1. Introduction

The solution to models that incorporate both idiosyncratic and aggregate risk is of central interest to any researcher seeking, for example, a quantitative model of the macroeconomy that can capture the effect of policy on inequality. Two pioneering papers demonstrated that these models—which feature a state vector that is typically not finite-dimensional, or even countably dimensional—are computable: den Haan (1997) and Krusell and Smith (1998). In particular, the second paper has opened a large literature that applies their result—that predicting future prices is possible using only the mean of the wealth distribution—to a wide variety of environments.[1] I describe in this paper a solution algorithm very close to Krusell and Smith (1998) with an important difference in the simulation procedure.

Before proceeding to the solution method, I want to make several points regarding terminology. The first is in regards to the term *approximate aggregation*, which is used by Krusell and Smith (1998) to describe their results. There are two ways to interpret this term. First, one could read approximate aggregation as implying that the economy's statistical properties are similar to an economy that can be solved using a stand-in household that chooses aggregates directly; that is, approximate aggregation means that there exists an agent whose decisions approximately coincide with aggregates. Second, one could make the weaker statement that approximate aggregation only implies that forecasting prices only requires first moments. Many economies approximately aggregate in the second sense but generate dynamics that are not

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[1] Good surveys can be found in Ríos-Rull (2001), Young (2007a), and Krusell and Smith (2006). The number of papers applying this insight is now very large.

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at all similar to an economy with a stand-in household, at least not one with the usual preferences.[2] Therefore, when one states that an economy approximately aggregates one must supply evidence that additional state variables do not change the dynamics of interest.

A second point involves the interpretation of the algorithm itself. The literature seems to have adopted a *bounded rationality* interpretation, in which the agents in the model are assumed to ignore some information about the economy. Presumably, this information would be useful but nevertheless ignored. However, Young (2007a) demonstrates that the information that is ignored (namely, all the higher-order moments of the wealth distribution) is simply not useful for forecasting prices; that is, agents are not boundedly rational at all. Thus, an equally acceptable (or perhaps more acceptable) interpretation of the algorithm is based on projection: the true doubly infinite-dimensional operator is approximated by projecting it onto a space of low dimension. This interpretation may be preferable because it implies that agents are not behaving suboptimally in any sense, so welfare calculations are well-defined.

2. Algorithm

The difficulty with solving a model with incomplete markets and aggregate uncertainty is that the entire distribution is part of the state of the world. While the current prices are only functions of the mean and the current aggregate shock, future prices could depend nontrivially on higher-order moments through nonlinearities in the individual decision rules. Since a continuous distribution is impossible to represent completely on a computer except in special cases, and even a discrete distribution might suffer severely from the curse of dimensionality, one must in general reduce the state space. As noted above, the approach here is to assume that households do not use all the information contained in the current state space to forecast future prices; however, in the model computed here (den Haan et al., 2009) no information that is useful for predicting future prices is being ignored.

2.1. Forecasting functions

To begin, I assume that the set of moments used to forecast future prices is given by M; denote the cardinality of this set by $\#M<\infty$. Since aggregates are only states because they help forecast prices—either current or future—the set of aggregate state variables is effectively reduced to only M. Note that m_1, the first moment of the capital distribution, must be an element of M since the current prices are functions of it. I then seek an "approximate equilibrium" law of motion $\widehat{H}:\mathbb{R}^{\#M}\rightarrow\mathbb{R}^{\#M}$ which maps current elements of M into their values next period, because the true law of motion H (which maps distributions into distributions) is infinite-dimensional and therefore not computable. Whether this equilibrium law of motion is adequate then depends on how well agents could do by using additional information into their forecasting functions or by using a different functional form; from the perspective of the households this additional information is freely available. It has been found that m_1 is itself nearly sufficient to predict the future evolution of prices if the law of motion is assumed to be log-linear:

$$\widehat{H}(m_1,a)=\exp(A(a)+B(a)\log(m_1)).$$

Note that the coefficients depend on the current aggregate shock a.[3] The Inada conditions for the firm ensure that aggregate capital remains positive and the logarithms stay real-valued.[4]

The success of the finite forecasting approach is discussed in Krusell and Smith (1998) and Young (2007a). The reason that the mean is nearly sufficient by itself is that agents have linear savings rules over most of the state space, and these rules are shifted in parallel by movements in the aggregate state (m_1,a). For the small measure of households who have nonlinear decision rules, their lack of wealth makes them irrelevant for determining aggregate capital. Thus, the mean actually is a nearly sufficient statistic for next period's return to capital. It is important to note that the mean is not a sufficient statistic for all aggregates, as higher-order moments of the wealth distribution are orthogonal to the mean and

[2] See Krusell and Smith (1998) for a demonstration of this result using an economy with idiosyncratic shocks to the discount factor. Young (2007a) shows that estimating the Euler equation of a stand-in household on artificial data generated from this model would produce estimates for risk aversion that are strongly biased downward, even in large samples.

[3] An alternative is to assume that a is simply another linear term:

$$\widehat{H}(m_1,a)=\exp(A+B\log(m_1)+Ca).$$

For this model it makes no difference which specification is used because B turns out not to vary with a. For some models that may not hold, in which case the more general version in the text would be needed.

[4] I also compute the model using the log-polynomial function

$$\widehat{H}(\mu_1,a)=\exp\begin{pmatrix}A(a)+B(a)\log(m_1)\\ +C(a)[\log(m_1)]^2\end{pmatrix}.$$

For several variants, this formulation was numerically unstable due to collinearity between $\log(m_1)$ and $[\log(m_1)]^2$ and was not computed. In any case, the presence of the quadratic term did not seem to affect the solution. An implementation using Chebyshev polynomials and/or increasing the number of observations is likely to mitigate the collinearity problem.