ELSEVIER Journal of Mathematical Economics 33 (2000) 441–448

JOURNAL OF Mathematical ECONOMICS

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A remark on rational expectation equilibria with incomplete markets and real assets

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Received 27 April 1998; received in revised form 14 June 1999; accepted 17 June 1999

Abstract

In this note, I established the existence, for a generic set of endowments, of a fully revealing rational expectation equilibrium (REE) in an economy characterized by incomplete markets and real assets. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Incomplete markets; Asymmetric information; Rational expectations; Real assets

1. Introduction

Models in which asymmetric information matters and in which agents form rational expectations were extensively studied since the seminal paper of Radner (1979). This approach was often applied to the case of a standard Walrasian model (see for instance, Allen, 1981, Anderson and Sonnenschein, 1982, Jordan, 1983...). But if one refers to Radner's paper, one notices that his contribution explicitly takes into account asset markets and even assumes incomplete markets. The great developments of this last field should therefore allow some deeper insights into the question of the existence of fully revealing rational expectation equilibria (REE for short).

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PII: S0304-4068(99)00025-7

In this line of research, Polemarchakis and Siconolfi, 1993 show that noninformative REE exist in models with nominal assets. In their case, the degree of indeterminacy of equilibrium prices and, generically, allocations is sufficient to prevent any revelation of information. The existence of indeterminacy moreover requires a refinement of the definition of a revealing REE. This one cannot be simply viewed as a one-to-one price selection with respect to aggregated information but also requires at least that an equilibrium price in one state of information does not belong to an equilibrium price set in another state. With this stronger definition motivated by the indeterminacy, fully revealing REE do not even exist.

At contrario, if the equilibrium is determined, strong REE are generically equivalent to REE and one can conjecture that revealing REE exist. This follows directly from Radner's one commodity model with real assets (see also Stahn, 1996 who adapted this model to a more standard approach in incomplete markets). It thus remains to be shown that the result is maintained if several commodities are available in each state. But in this instance, two situations must be studied: the case of numeraire and real assets (see respectively Geanokoplos and Polemarchakis, 1986; Duffie and Shafer, 1985). The first case was studied by Pietra and Siconolfi (1998). They showed that all financial asset REE are revealing for a generic subset of the endowment space. In this note, I will show the generic existence of a fully revealing REE in the case of real assets.

The technique is quite standard. I introduce the definition of a full communication equilibrium (na-FCE) characterized by no-arbitrage and verify that it coincides with a revealing REE as long as the prices are different in each state of information and the return matrices are, in each state, of full rank. Having noted this, it remains to be established that a na-FCE satisfies these properties for a generic subset of the endowment space.

This paper is organized as follows: In Section 2, I describe the model, Section 3 is devoted to the definition of an REE and of a na-FCE, the main steps of the proof are given in Section 4. The remaining steps are established in Section 5.

2. The model

Let me consider an economy over two periods ($t = 0,1$) and S states at date 1. Each state is indexed by $s \in S = \{0, \ldots, S\}$, state $s = 0$ stands for period 0 and $S^* = S \setminus \{0\}$. Commodities, $\ell \in L = \{1, \ldots, L\}$, are traded at each date by the consumers, $i \in I = \{1, \ldots, I\}$, and real assets, $k \in K = \{1, \ldots, K\}$ in number $K < S$, are available at date 0. Each agent $i \in I$ observes, at date 0, a private information $m_i \in M_i = \{1, \ldots, M_i\}$ which is a part of the state of information $m = (m_i)_{i \in I} \in M = \prod_{i \in I} M_i = \{1, \ldots, M\}$. This state is known at date 1. The information available to agent i is described by a partition $\mathcal{I}_i = \{\iota_\lambda\}_{\lambda \in \Lambda}$ of M. $\mathcal{I}_i(M_i)$ denotes the partition induced by his private information.