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Overlapping generations models with incomplete markets

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Abstract

This paper addresses existence of equilibria for an overlapping generations exchange economy with incomplete markets, one-period real assets and bounded short sales, defined on an infinite event-tree where there is a continuum of branches at each node of the tree.

The demographic structure used in the paper is quite general and allows for infinite-lived as well as finite-lived agents. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In this paper, we model overlapping generations of agents on an infinite event tree with a continuum of branches at each node. Markets are sequentially incomplete: there are finitely many one-period real assets traded at each node. Our work can be seen as a merger of the overlapping generations model with finitely many states at each node and the incomplete markets model with finitely many agents and a continuum of states.

Stochastic overlapping generations models were first studied by Lucas (1972), have ever since been used in macroeconomics and provided a useful setting for the analysis of sunspot equilibria (see Azariadis (1981), Shell (1977), Spear (1984)). When there are only finitely many states at each node, existence can be established under assumptions that are similar to those guaranteeing existence in a deterministic overlapping generations model,

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namely, for finite lifespans or under conditions that ensure that the aggregate endowment is a bounded sequence (see Geanakoplos and Polemarchakis (1991) on the deterministic case and Schmachtemberg (1988) on the incomplete markets case). Most of the literature on equilibria in this class of stochastic models has been devoted to two consequences of the introduction of uncertainty: the increase in the dimension of indeterminacy and the difficulties in obtaining stationarity of equilibrium. When there is more than one good stationary equilibria may not exist (see Spear (1985), Spear (1988), Spear and Srivastava (1986)). In single-good economies, particular attention has been paid to existence of stationary equilibria with nonzero prices of assets yielding no dividends (see Cass et al. (1992), Gottardi (1996) on monetary equilibria and Demange and Laroque (1999) on social security transfers).

Introducing a continuum of states created an important difficulty for the existence of equilibria in incomplete markets with finitely many agents. In fact, equilibria was shown to exist under the indispensable condition that ex-post endowments are bounded from below. That is, asset returns should be relatively small, relative to endowments, so that, for any portfolio choice and any spot prices, in case of a negative financial income, endowments' income is always enough to pay the debt. For the finite horizon case, existence was studied by Mas-Colell and Monteiro (1996), Hellwig (1996), Mas-Colell and Zame (1996), Monteiro (1996). For the infinite horizon case, existence was established by Araujo et al. (1996) under additional assumptions on bounded short-sales and severe discounting. The indispensable assumption can be dropped when addressing existence of equilibria with default (see Araujo et al. (1996), Araujo et al. (1998)).

Our results on existence of equilibrium in overlapping generations models with a continuum of states require, on the one hand, the conditions assumed in models with finitely many agents and a continuum of states and, on the other hand, the usual conditions imposed in overlapping generations models. First, short-selling constraints are assumed to be stringent enough (or return coefficients to be small enough) in order to guarantee that ex-post incomes are positive and bounded away from zero. This hypothesis was used in all the above papers on existence of equilibrium with incomplete markets and a continuum of states, except Araujo et al. (1998) where a different equilibrium concept, allowing for default, was addressed. Secondly, to allow for the case of agents with infinite life, we assume for these agents the same sufficiently severe discounting as in the infinite horizon model by Araujo et al. (1996) and we require also that the aggregate endowment is uniformly bounded, over nodes. It is well known that, even in the deterministic overlapping generations models, the unboundedness of aggregate endowment may create serious difficulties for existence of equilibrium (see Geanakoplos and Polemarchakis (1991)). The assumption is satisfied under Wilson (1981) condition that a finite set of agents owns a non-negligible fraction of aggregate endowment. It is also satisfied under an alternative condition bounding utilities from above and marginal utilities from below. We do not address stationarity of equilibria.

To prove the existence of equilibrium, we combine a finite truncation of the horizon with finite-dimensional approximations of utilities, endowments and asset returns. Each truncated economy has an equilibrium and each consumer's maximization problem satisfies Slater's condition. As in Araujo et al. (1996), establishing uniform upper bounds for the Kuhn–Tucker multipliers associated with these optimization problems plays a crucial role in the equilibrium existence proof. However, our argument for establishing these bounds