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Solving the incomplete markets model with aggregate uncertainty using parameterized cross-sectional distributions

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ABSTRACT

This note describes how the incomplete markets model with aggregate uncertainty in Den Haan et al. [Comparison of solutions to the incomplete markets model with aggregate uncertainty. Journal of Economic Dynamics and Control, this issue] is solved using standard quadrature and projection methods. This is made possible by linking the aggregate state variables to a parameterized density that describes the cross-sectional distribution. A simulation procedure is used to find the best shape of the density within the class of approximating densities considered. This note compares several simulation procedures in which there is—as in the model—no cross-sectional sampling variation.

1. Introduction

This paper describes the algorithm used to solve the model with incomplete markets and aggregate risk of Den Haan et al. (2009). The algorithm of Krusell and Smith (1998), the most popular algorithm to solve this type of model, consists of an iterative procedure and in each iteration a simulation of the economy with the approximating solution is used to solve for the law of motion of aggregate capital. The simulation procedure of Krusell and Smith (1998) has two types of sampling variation. The first is due to using a finite instead of a continuum of agents. As shown below, this sampling variation can be avoided. The sampling variation that is due to the aggregate shock, however, seems unavoidable. Using simulated data to obtain numerical solutions has two disadvantages. First, by introducing sampling noise the policy functions themselves become stochastic. This effect can be reduced by using long time series, but sampling noise disappears at a slow rate. Second—and more importantly—the values of the state variables used to find the best fit for the aggregate law of motion are endogenous and are typically clustered around their means. But accuracy can be improved by using values that are more spread out.[1] In particular, the numerical literature advocates the use of Chebyshev nodes to ensure uniform convergence and the procedure used here allows for this efficient choice of grid points.

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[1] Recall that the standard errors of regression coefficients, $\sigma^2(X'X)^{-1}$, are lower when the x-values are more spread out.

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The algorithm described here uses projection methods and can—in principle—solve the model without relying on any simulation procedure. Using projection procedures to solve a model with a continuum of agents typically requires a parameterization of the cross-sectional distribution as in Den Haan (1997).[2] We improve on the procedure proposed in Den Haan (1997) in the following way. If one parameterizes the cross-sectional distribution, then all parameters of the density are state variables. For example, if one uses a Normal density then there are two parameters, i.e., the mean and the variance, and thus two state variables.[3] But note that using a Normal density has implications for the higher-order moments. These implied higher-order moments may not be correct. For example, a Normal density implies no skewness, but the model one tries to solve may have a skewed distribution. In that case one could allow for more general approximating functions with more free parameters. The problem of adding coefficients to the approximating density is that one also adds state variables. Our procedure uses an approximation for the density that allows for more flexibility, but does *not* increase the number of state variables.

The idea is the following. Suppose one starts with the Normal as the approximating cross-sectional density and uses the mean and the variance as state variables. Using this approximating density one can obtain a numerical solution of the model using standard projection methods and without any simulation. Now that one has obtained a numerical solution one can ask the question whether the cross-sectional density is described accurately with a Normal density. To answer this question one has to rely on a simulation. Suppose that after simulating a panel and calculating the higher-order (unconditional) cross-sectional moments, one concludes that the Normal does not provide an accurate representation. When using the algorithm of Den Haan (1997), one would use a higher-order approximation of the cross-sectional distribution and increase the number of state variables.

But one can also modify the functional form of the cross-sectional distribution without adding state variables. This is the approach followed here, that is, the information obtained from the simulation is used to modify the functional form of the cross-sectional distribution. Thus, if the Normal is not accurate one would use at each point on the grid a density that (i) implies values for the higher-order moments equal to the values found in the simulation and, of course, (ii) implies values for the lower-order moments that are included as state variables. The algorithm iterates on this procedure until the information provided by the simulation is consistent with the assumptions made about the shape of the cross-sectional distribution. The philosophy that underlies our algorithm is similar to the one in Reiter (2009). The differences are mainly in terms of implementation, which is less cumbersome for our algorithm.

Although we rely on a simulation procedure, it plays a much smaller role than in, for example, the algorithm of Krusell and Smith (1998); it is only used to determine the shape of the density. The procedure to solve for the policy rules uses standard projection techniques without a simulation step.

Algan et al. (2008) (AAD hereafter) propose a new procedure to simulate cross-sections with a continuum of agents. The most common procedure to simulate models with a continuum of agents consists of using a finite number of agents and a random number generator to draw the idiosyncratic shocks. Consequently, the results are subject to cross-sectional sampling variation. Models with a continuum of agents do not have this property and most solution procedures are based on this lack of sampling variation.[4] AAD show that sampling variation can be substantial and that properties of the laws of motion may be overlooked because of the presence of cross-sectional noise. In this note, we compare three procedures that all avoid cross-sectional sampling variation.

2. Algorithm

This section provides an overview of the key ingredients of the algorithm.[5]

Projection method: The numerical solution of the incomplete markets model with aggregate uncertainty in Den Haan et al. (2009) consists of a policy function $k'(\varepsilon, k, a, s; \Psi_k)$, where ε is the (exogenous) individual employment status,[6] k the individual capital stock, a the exogenous aggregate state, s a set of variables that characterizes the cross-sectional joint distribution of capital and employment status, and Ψ_k the coefficients of the policy function. The variable s refers to the beginning-of-period distribution after the new employment status has been observed.

The standard projection procedure to solve for Ψ_k consists of the following three steps.

1. Construct a grid of the state variables.

[2] Den Haan and Rendahl (2009) show that aggregation *without* explicit distributional assumptions is possible when the individual policy functions are linear in the coefficients. They implicitly obtain information about the distribution by approximating auxiliary policy rules.

[3] As shown below, one can establish a mapping between the parameters of the approximating density and a set of moments even if more flexible densities are used. Instead of using the parameters of the density, we always use moments as state variables.

[4] For example, solution procedures typically specify that next period's distribution is fully determined by the current distribution and aggregate shocks.

[5] A more in depth discussion can be found in AAD.

[6] The value of ε is equal to 0 when the agent is unemployed and equal to 1 when the agent is employed.