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Rothschild–Stiglitz's definition of increasing risk and the relationship between volatility and risk premium

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Abstract

This paper analyzes the relationship between volatility and risk premium under the capital asset pricing model and Rothschild and Stiglitz's [Rothschild, M. and J.E. Stiglitz. (1970) Increasing risk I: a definition. Journal of Economic Theory, 2, 225–243.] definition of increasing risk. Especially examined are the conditions of the widely used assumption of constant correlation, which results in a linear relationship. Though both the above model and definition are widely known and accepted, their compatibility has remained unclear in the literature. According to this paper, they are in harmony with the linear relationship, if the correlation between a stock and the market portfolio is less than 0.7. Otherwise a conflict may arise.

JEL classification numbers: C61; D80; E22; G00; G31
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1. Introduction

It is commonly accepted that the risk premium increases with systematic risk. However, the relationship between total risk, measured by volatility, and the risk premium is not clear. Depending on the field of study, the literature contains widely varying assumptions about the effect of volatility on the risk premium. Many studies on absolute pricing assume that volatility increases the risk premium. On the other hand, studies on derivative asset pricing often implicitly assume that the risk premium does not depend on volatility.

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The relationship between changes in volatility and risk premium is very complex. In reality, both may simultaneously increase, they may be independent of each other, or even opposite. First, suppose that a firm faces a specific risk that does not affect other firms, for example, that a key official is going to leave the firm. Clearly, the variance of the firm's future profits increases. Though total risk increases now, the risk premium remains unaffected because this particular firm's specific risks can be eliminated from the portfolio through diversification. Second, consider, for example, the stock of a gold company, whose price understandably depends on the price of gold. Technically, the gold price is the state variable of the stock price, and the stock can thus be seen as a derivative instrument on gold. As I show in Appendix A, the volatility of the stock may now vary as the stock price or the gold price changes or as time passes in such a way that the risk premium changes linearly with volatility under the Capital Asset Pricing Model (CAPM). An increase in the risk premium may also be caused endogenously. Managers can control volatility and the risk premium, for example, with the amount of debt the company has. If they increase the debt-equity ratio, both volatility and the risk premium increase, the change being endogenous. Appendix A also demonstrates that if the gold price is modelled as a mean-reverting process, the risk premium may change linearly with volatility as volatility responds to a change in the company's debt level or production costs. It is important to note that Appendix A provides examples only of a linear relationship, which cannot be generalized because, as mentioned above, all kinds of relationships are possible.

Though it is not reasonable to assume that volatility always increases the risk premium, such an assumption is very common in the literature. More specifically, based on the CAPM, the risk premium is often assumed to increase linearly with volatility so that the correlation between an asset and the market portfolio is kept fixed. For example, Merton's (1973) monumental paper on the intertemporal capital asset pricing model assumes a constant correlation (see footnote 18 in Merton (1973, p. 873)). In addition, many stochastic volatility models rely on the constant correlation assumption, and Pagan (1996) among others considers it plausible: "it does not seem unreasonable to restrict the conditional covariances to vary in line with the conditional variances." In fact, following Bollerslev (1990), the standard practice in modelling asset return dynamics is to assume a constant correlation between asset and the market portfolio (see, for example, Kroner & Ng 1998; Kroner & Sultan 1993; Ng 1991; Zhu 2002). Furthermore, the relationship between volatility and the risk premium is often directly assumed to be linear (see, for example Duan 1995). Besides the constant correlation model, other widely used time-varying covariance models comprise the VECH model of Bollerslev, Engle, and Wooldridge (1988), the factor ARCH model of Engle, Ng, and Rothschild (1990), and the BEKK model of Engle and Kroner (1995). These models do not limit covariance to be relative to volatility.

While studies on absolute pricing usually characterize the relationship as positive, studies on derivative asset pricing typically implicitly assume that volatility does not affect the risk premium. We may note that the risk premium is an "invisible" parameter in option prices — the risk premium does not appear in derivative pricing equations but affects option prices via the stock price. If the risk premium increases with volatility, the cash-flows of the underlying project are discounted at a higher rate, leading to a decrease in the present value of the project and an increase in dividend yield. In fact, while this holds, the effect of volatility on a call price may be negative, because opposing effects may appear via the present value of the underlying asset and the dividend yield. For the opposite effect to come into play requires only that the risk premium increase in volatility. Usually, this invisible relationship between volatility and risk premium does not appear in the derivative pricing literature, but some studies have recognized it. Sarkar (2000) found that an increase in volatility can actually increase the probability of investment, a finding contrasting with earlier predictions. Before this, McDonald and Siegel (1985) noted that an option