

Influence of saving propensity on the power-law tail of the wealth distribution

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Abstract

Some general features of statistical multi-agent economic models are reviewed, with particular attention to the dependence of the equilibrium wealth distribution on the agents' saving propensities. It is shown that in a finite system of agents with a continuous saving propensity distribution a power-law tail with Pareto exponent $\alpha = 1$ can appear also when agents do not have saving propensities distributed over the whole interval between zero and one. Rather, a power-law can be observed in a finite interval of wealth, whose lower and upper ends are shown to be determined by the lower and upper cutoffs, respectively, of the saving propensity distribution. It is pointed out that a cutoff of the power-law tail can arise also through a different mechanism, when the number of agents is small enough. Numerical simulations have been carried out by implementing a procedure for assigning saving propensities homogeneously, which results in a smoother wealth distributions and correspondingly wider power-law intervals than other procedures based on random algorithms.

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1. Introduction

Statistical mechanical models of closed economy systems have received considerable attention in recent years due to the fact that they seem to predict realistic wealth and income distributions shapes from a simple underlying dynamics, similar to that of microscopic models of classical statistical mechanics [1–27]. For an overview of the current situation of these models see Ref. [27].

In fact, as found in empirical distributions, they can reproduce a Boltzmann distribution at intermediate and a power-law at higher values, see e.g. Refs. [28–32]. A power-law form in the tail of statistical distributions was observed more than a century ago by the economist Pareto [33], who found that the wealth of individuals

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in a stable economy has a cumulative distribution $F(x) \propto x^{-\alpha}$, where α , the Pareto exponent, has a value between 1 and 2.

In this paper, we consider models in which N agents interact exchanging a quantity x , that can be interpreted as a measure of the agents' wealth, expressed in money units. Depending on the parameters of the model, in particular on the values of the saving propensities $\{\lambda_i\}$ ($i = 1, \dots, N$) of the N agents, the equilibrium wealth distribution can be a simple Boltzmann distribution for $\lambda_i = 0$ [3,4,9], a Gamma distribution with a similar exponential tail but a well-defined mode $\bar{x} > 0$ for $\lambda_i = \lambda_0 > 0$ [1,2,6,10,14,15], or a distribution with a power-law tail for randomly distributed λ_i [16,18]. It has been recently recognized [18,23] that the observed power-law arises from the mixture of Gamma distributions corresponding to agents with different values of λ . That is, in systems where the saving propensity is distributed according to an arbitrary distribution function $g(\lambda)$, individual agents relax toward a Gamma distribution similarly to systems with a global saving propensity λ_0 , with the important difference that in this case the various Gamma distributions corresponding to different λ 's will mix in such a way to produce a power-law.

In order for the power-law $F(x) \propto x^{-1}$ to be produced, a special role is played by agents with values of the saving propensity close to $\lambda = 1$: altering this part of the λ -distribution can strongly modify the tail of the wealth distribution [18]. The aim of the present paper is to further investigate quantitatively this important point, by studying in general terms the relation between the saving propensity distribution and the wealth distribution tail. We begin in Section 2 by recalling the main features of statistical multi-agent models. In Section 3 we focus on the relation between saving propensity and wealth distribution. First, we show that when the number of agents is low, discreteness effects may show up as an upper cutoff of the wealth power-law tail. Then through numerical simulations we illustrate how—even when the number of agents is high—the lower and upper cutoff of the saving propensity distribution determine those of the wealth distribution power-law. This is further discussed with some examples of realistic wealth distributions. Results are summarized in Section 4.

2. Statistical multi-agent models

In statistical multi-agent models N agents interact with each other through pair interactions in which a quantity x , generally referred to as “wealth” in the following, is exchanged. Each agent i ($i = 1, \dots, N$) is characterized by the wealth x_i and, possibly, by some parameters, such as the saving propensity λ_i . The time evolution of the system is carried out by extracting randomly at every time step two agents i and j , who exchange an amount of wealth Δx between them,

$$\begin{aligned} x'_i &= x_i - \Delta x, \\ x'_j &= x_j + \Delta x. \end{aligned} \quad (1)$$

It can be noticed that in this way the quantity x is conserved during the single transactions, $x'_i + x'_j = x_i + x_j$, where x'_i and x'_j are the agent wealths after the transaction has taken place.

2.1. The basic model

In a basic version of the model Δx is assumed to have a constant value [3–5],

$$\Delta x = \Delta x_0, \quad (2)$$

or to be proportional to the initial wealths [1,2,9],

$$\Delta x = \bar{\varepsilon}x_i - \varepsilon x_j, \quad (3)$$

where ε is a random number uniformly distributed between zero and one and $\bar{\varepsilon} = 1 - \varepsilon$. The form of Δx defined by Eq. (3) produces a random reshuffling of the total wealth, given by the sum of the wealths of the two agents [9], since Eq. (1) can be rewritten as

$$\begin{aligned} x'_i &= \varepsilon(x_i + x_j), \\ x'_j &= \bar{\varepsilon}(x_i + x_j). \end{aligned} \quad (4)$$

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