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Double power laws in income and wealth distributions

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Abstract

Close examination of wealth distributions reveal the existence of two distinct power law regimes. The Pareto exponents of the super-rich, identified, for example in rich lists such as provided by Forbes, are smaller than the Pareto exponents obtained for top earners in income data sets. Our extension of the Slanina model of wealth is able to reproduce these double power law features.

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1. Introduction

The first person to study the topic of wealth distributions in a quantitative manner, Pareto, was trained as an engineer [1]. In recent years, it is the physics community that has made significant contributions to the topic, again by focusing not only on theoretical methodologies [2–6] but also making comparisons of their results with empirical data [7–11]. For a recent detailed review of the subject see Ref. [12].

What does seem clear from the mounting evidence is that income and wealth distributions across societies everywhere follow a robust pattern and that the application of ideas from statistical physics can provide understanding that complements the observed data. The distribution rises from very low values for low income earners to a peak before falling away again at high incomes. For very high incomes it takes the form of a power law as first noted by Pareto. The distribution is certainly not uniform. Many people are poor and few are rich.

The cumulative probability, $P(_{>}m)$ corresponds to the probability of finding earners that have an income bigger or equal to a certain amount of income, m. For values of m less than the average income it decreases slowly from its maximum value 1. For values roughly higher than the average it follows a power law

$$P(_{>}m) = m^{-\alpha} \tag{1}$$

where α is the Pareto exponent.

Looking closely at results for income and wealth distributions around the world (Table 5.2 in Ref. [12]) we see that the values for the exponents for wealth/income data sets and data that concern only the top wealthiest people in

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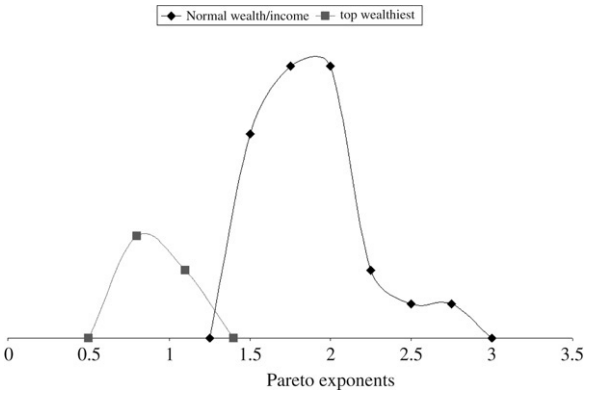

Fig. 1. Distribution of the Pareto exponents found by different authors in the last decade. The black curve is from datasets taken from tax/income databases. The grey curve is from super-rich lists, such as Forbes. The Pareto exponent for the top richest is around one, while for the "normal" rich people it is around two (data taken from Table 5.2 in Ref. [12]).

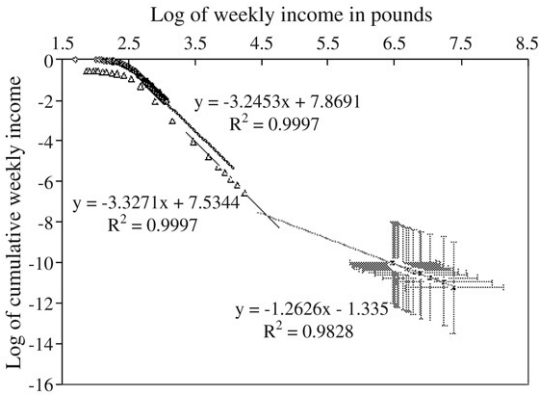

Fig. 2. Distribution of the cumulative weekly income in UK for 1995. Data for income $<10^{4.5}$ pounds represents income for 1995 from UK Revenue Commissioners and UK New Income Survey and a similar Pareto exponent is achieved for the high end of these curves, ∼3.2–3.3. The data for income $>10^{5.5}$ pounds represents an estimation of the income, in 1995, for the top richest in UK. In this case the Pareto exponent is lower and around 1.3.

society differ. Fig. 1 shows the distribution of the Pareto exponents when we take these different origins of the data into account. The average Pareto exponent is approximately 2.0 for the top earners in tax/inheritance statistics, and just 1.1 for the super-rich. Using test statistics to compare the differences between both samples we reject the hypothesis of there being no difference between the mean of both populations.

We believe that the studies of wealth that are based on tax/income generally do not include the wealth of very rich people. A further indication of two power law regimes is the study of Souma [7]. In Figure 1 of his paper [7], Souma found a Pareto exponent of 2.06 in the high end. However, we see an indication of a second power law for the top richest (higher than 3000 million yen) which we estimate as an exponent below 1.0 based on his figure. Yet a further indication of two power laws comes from our analysis of UK data. In Fig. 2 we show data for the cumulative distribution of incomes in the UK for the year 1995. The upper curve is calculated from survey data and tends to a power law which was confirmed by data obtained by Cranshaw [13] from the UK Revenue Commissioners. The lower curve is calculated using the UK New Income Survey data, which takes a 1% sample of all employees in Great Britain. The slight shift in the two curves is due to uncertainty in a normalization factor but the power law is clearly seen and extends from weekly incomes of just under £1000 per week up to around £30 000 per week. Over this region the Pareto exponent is ∼3.3. This might be assumed to be the end of the story with the power law being associated with Pareto's law. However, from data published by Sunday Times [14] for the wealth of billionaires in UK for 2006, we can make an estimate of the income in 1995 generated by the wealth. In order to move from 2006 back to 1995, we made some creative estimations. First, we said that probably the wealth of the top richest group had increased in