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Statistical Equilibrium Wealth Distributions in an Exchange Economy with Stochastic Preferences

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We describe an exchange market consisting of many agents with stochastic preferences for two goods. When individuals are indifferent between goods, statistical mechanics predicts that goods and wealth will have steady-state gamma distributions. Simulation studies show that gamma distributions arise for a broader class of preference distributions. We demonstrate this mathematically in the limit of large numbers of individual agents. These studies illustrate the potential power of a statistical mechanical approach to stochastic models in economics and suggest that gamma distributions will describe steady-state wealths for a class of stochastic models with periodic redistribution of conserved quantities. *Journal of Economic Literature* Classification Numbers: C15, C62, C73, D3, D5. © 2002 Elsevier Science (USA)

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1. INTRODUCTION

Academic models of financial markets since the beginning of the 20th century have noted the essentially random nature of successive changes in

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asset prices. Yet classical economic theory describes deterministic transitions of the detailed states of an economic system which balance supply and demand in response to changes in preference and in information (e.g., in expectations of future returns). This dichotomy between (largely) deterministic detailed transitions and stochastic observed aggregates in economics [1, 2] is analogous to that in physics between motions of Newtonian particles and macro- level observations of Brownian motion and thermodynamic variables [3–5]. In both economics and physics, the limiting behavior for superposed effects of large numbers of individuals (particles) is due to probabilistic central limit theorems. The first such theoretical results asserted Gaussian-distributed displacements [1–4]. More realistic models in physics, going beyond Gaussian observations, have incorporated interactions among particles [5]. In economics, statistical description of aggregates based on probabilistic limit theorems has been broadened to include infinite-variance laws and stable-increments processes [6–8]. Economic models leading to non-Gaussian distributions often involve complex or heterogeneous interactions among market participants [9–11], but results with true statistical-mechanics flavor, deriving such behavior from detailed microeconomic-level mechanisms, are rare.

One early attempt to incorporate thermodynamic and statistical mechanical reasoning in economics [12] was largely confined to analogy without detailed mathematical content. Another approach [13], which we largely follow, formulates the problem of economic market equilibrium as that of finding a time-invariant, nondegenerate probability distribution in a high-dimensional state space describing the fortunes of a large population of firms and individuals. The authors of [13] propose that microeconomic production decisions be treated deterministically while market fluctuations based on changing preferences and exogenous influences be considered a stochastic process. Reference [2] provides one of the few cases of analysis with simplifying large-population approximations leading to true invariant distributions of a large state-space Markov model in economics. There are several references, and a growing physics literature, with statistical mechanics flavor in economic modeling, which describe equilibria via maximum "entropy" configurations subject to constraints such as fixed total wealth [14–16].

In this paper, we adopt a combined theoretical and simulation approach to a simple model in which a fixed supply of goods is traded among individuals whose preferences for goods are determined stochastically. A central feature of our model is that individuals periodically change their preferences for goods independently, according to a distribution of preferences in the population. The changes in aggregate demand translate in the next time-period into changes in the market-clearing price. As individuals adjust the mix of assets they hold to satisfy their changing