The welfare consequences of strategic behaviour under approval and plurality voting

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A B S T R A C T

This paper studies the welfare consequences of strategic behaviour under approval and plurality voting by comparing the utilitarian efficiencies obtained in simulated voting under two behavioural assumptions: expected utility-maximising behaviour and sincere behaviour. Under approval voting utilitarian efficiency is relatively high irrespective of the behavioural assumption, and under the plurality rule strategic voting significantly increases utilitarian efficiency.

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1. Introduction

This paper investigates the welfare consequences of strategic behaviour under approval voting (AV) by comparing utilitarian efficiencies obtained with *Expected Utility maximising voting behaviour* (EU behaviour) and with *Sincere Voting behaviour* (SV behaviour). Under SV behaviour voters are assumed to vote for all those candidates for which the utility exceeds the average for all candidates (Merrill, 1979; Brams and Fishburn, 1983, p. 85; Ballester and Rey-Biel, 2007). Under EU behaviour voters give their votes to different candidates depending on expected-utility calculations (Merrill, 1981a,b). They give a vote to a candidate under EU behaviour if the expected gain from doing so is positive. The difference between EU and SV behaviour under AV is thus that the voters are engaged in probability calculations in the former but not in the latter (see e.g., Niemi, 1984).[1]

A voter is usually defined as voting sincerely under AV if he or she gives a vote to all candidates standing higher in his or her ranking than the lowest-ranking candidate for whom he or she gives a vote. There are no 'holes' in a voter's approval set.[2] If insincere voting is defined as not voting sincerely, it is commonly considered to be rare under AV. In contrast, *strategic behaviour* is not rare under AV, and the focus of this paper is on the welfare consequences of such behaviour rather than insincere or 'strategic' voting. The welfare consequences of *strategic voting* under AV are thus not studied here, if it is defined by the fact that a voter gives his or her vote to a candidate who is lower in his or her ranking than some candidate for whom he or she does not vote (see e.g., Brams and Sanver, 2006).

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[1] Although Brams and Fishburn (1983, p. 85) use an expected-utility terminology, their mean utility rule is classified as sincere here.
[2] See e.g., Brams and Fishburn (1978, 1983, p. 29) and Brams and Sanver (2006).

Table 1
Voters types and utilities

Type of voters						
t_1	t_2	t_3	t_4	t_5	t_6	U^i
x	Y	z	x	y	z	U^i_1
y	Z	x	z	x	y	U^i_2
z	X	y	y	z	x	U^i_3

Utilitarian efficiency is defined as the percentage of simulated voting games in which the candidate that maximises the sum of voters' utilities (the utilitarian winner) is selected (e.g., Merrill, 1988). The main finding is that utilitarian efficiencies are high under AV irrespective of the behavioural assumption used. Furthermore, given that the aim is to also evaluate whether it might be reasonable to introduce AV in mass elections, these efficiencies are compared to those under plurality voting (PV). It is shown that utilitarian efficiencies are higher under AV than under PV. Indeed, utilitarian efficiencies are higher than under any voting rule that has been studied with similar methods (see Lehtinen, 2006b, 2007a,b).

Brams and Fishburn (e.g., 1983, 2005) have presented various arguments for AV. I will try to clarify or modify at least two of them. One argument is that this rule takes information on preference intensities into account (Brams et al., 1988). AV differs from other commonly used voting rules in that it allows for expressing intensity information even with SV behaviour, and in that voters never need to abandon their most-preferred candidate when they engage in strategic behaviour (e.g., Brams and Fishburn, 2005). However, thus far this intensity argument has been based on the mere intuition that since approval information is closely related to intensity information, it may be expressed under AV. This paper provides a formal model with which this question can be explicitly studied. The findings provide confirmation that this argument is correct under both behavioural assumptions. On the other hand, they imply that given a utilitarian evaluation of outcomes, the beneficial features of AV do not depend on the somewhat questionable assumption that voters have 'dichotomous' preferences (Brams and Fishburn, 1983, Ch. 2–3): AV is the best rule in terms of reflecting preference intensities.[3]

It has also been claimed that AV makes strategic voting unnecessary (Brams and Fishburn, 1978), but the welfare effects of strategic behaviour are still considered somewhat controversial. As (Niemi, 1984) has argued (see also van Newenhizen and Saari, 1988a,b), even though strategic voting may be rare under AV, even sincere voting may require a considerable amount of strategic thinking under this rule. The findings reported here show that whether or not voters engage in strategic calculations, AV yields high utilitarian efficiencies and thus often selects candidates with broad public appeal (cf. Brams and Fishburn, 1983, pp. 135, 171).

Strategic voting increases utilitarian efficiency under various voting rules because it allows for expressing preference intensities (Lehtinen, 2006b, 2007a,b). These results depend on the counterbalancing of strategic votes: broadly accepted candidates are likely to obtain many strategic votes and to lose few. Given that the decision to give more than one vote under AV is analogous to giving a strategic vote under other voting rules, there is also counterbalancing under AV. *Counterbalancing of second votes* implies that broadly supported candidates obtain more second votes than candidates facing strong opposition. This explains why utilitarian efficiencies under EU behaviour are also high under this rule.

Voters' beliefs are derived by combining methods of computing pivot probabilities (Hoffman, 1982; Cranor, 1996) with a signal-extraction model similar to the one provided by Lehtinen (2006a), and to global games (Carlsson and van Damme, 1993; Morris and Shin, 2003). Voters obtain noisy signals of the true structure of the game and formulate beliefs on the basis of that.

Computer simulations are used for deriving the results because there is a large number of agents that are heterogeneous both with respect to their beliefs and to their preferences, and the aggregate-level outcomes depend on the voting interaction. Aggregating individual votes in an analytical model would be very difficult.[4]

The structure of the paper is the following. Optimal strategies under approval voting are formulated in Section 2. Section 3 introduces the model of incomplete information by describing the assumptions about voters' signals and beliefs. Section 4 describes the computer simulations. The simulation results are presented in Section 5. Section 6 shows that the results are robust with respect to different interpersonal comparisons of utilities. Section 7 presents the conclusions.

2. Strategic behaviour under approval and plurality voting

Let $X=\{x,y,z\}$ denote the set of candidates (with generic members j, k and m). The six possible types of voters and their preference orderings are presented in Table 1 below. U^i_k denotes voter i's payoff for the kth best candidate.

Under AV, voters give a vote to any number of candidates. Let $N=2001$ denote the total number of voters, and let n_j denote the number of voters who prefer candidate j the most. Given that the aim is to study mass elections, the maximum computationally feasible number of voters was selected. Changing the size of the electorate will be briefly considered later in discussing the simulation results.

Let n_j^{AV} denote the number of votes candidate j obtains under sincere behaviour under AV, and let n^{AV} denote the total number of votes cast under AV. Let v_x^{PV}, v_y^{PV}, and v_z^{PV} denote the *vote shares* of candidates x, y and z if all voters vote sincerely under PV:

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[3] This result seems to imply that modifying AV is unnecessary (see Yilmaz, 1999).
[4] See Lehtinen and Kuorikoski (2007) for a discussion on simulations in economics.