Generation of *bi-monotone* patches from quadrilateral mesh for reverse engineering

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ABSTRACT

Thanks to recent improvements, computational methods can now be used to convert triangular meshes into quadrilateral meshes so that the quadrilateral elements capture well the principal curvature directional fields of surfaces and intrinsically have surface parametric values. In this study, a quadrilateral mesh generated using the mixed integer quadrangulation technique of Bommes et al. is used for input. We first segment a quadrilateral mesh into four-sided patches. The feature curves inside these patches are then detected and are constrained to act as the patch boundaries. Finally, the patch configuration is improved to generate large patches. The proposed method produces *bi-monotone* patches, which are appropriate for use in reverse engineering to capture the surface details of an object. A *shape control* parameter that can be adjusted by the user during the patch generation process is also provided to support the creation of patches with good bi-monotone shapes. This study mainly targets shape models of mechanical parts consisting of major smooth surfaces with feature curves between them.

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1. Introduction

The ability to capture surface details of mechanical CAD objects accurately and robustly is still a major issue in reverse engineering. Compared to using general meshes, the adoption of parametric surfaces to represent objects provides a much more efficient and compact solution. Although many studies have focused on the use of triangular meshes to generate surface models, little work with quadrilateral meshes as input has been performed even though their utilization simplifies the conversion process. The elements of quadrilateral meshes are well arranged in such a way that they capture the principal curvature directional fields in a natural way, unlike triangular meshes. Moreover, quadrilateral meshes intrinsically have surface parameter values, enabling surface model generation with no parameterization step.

Using quad patches to represent an object is a popular approach today, because of their appealing properties (e.g., shape simplicity and ease of trimming) in surface modeling applications. The basic concept involves partitioning a quadrilateral mesh by removing irregular vertices, whose valences are fewer or more than four valences, so that all patches are four-sided and form a regular quadrilateral grid structure. Such partitioning is beneficial for surface fitting because parameterization can be achieved efficiently using a regular grid structure, and because there is no need for

surface trimming. However, it is not always possible to determine the feature curves of objects within the boundaries formed by such partitioning which leads to quality problems with the generated surfaces. To capture all feature curves with an appropriate number of segments (see Fig. 1(a)), it may be necessary to allow patches with arbitrary shapes. However, this is problematic because it results in an increased number of trimmed curves.
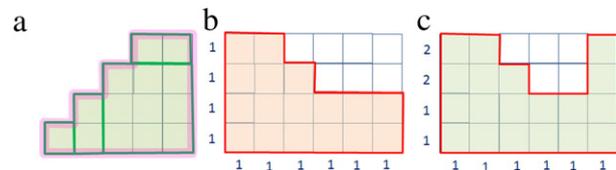
Fig. 1. (a) Four quad patches (in green) capture the shape, however same shape can be captured by a single arbitrary-shaped patch (in pink). (b) Bi-monotone patch, (c) non-monotone patch (numbers in blue shows the number of quadrilateral segments for rows or columns). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

We take the concept in between quad patches and patches with arbitrary shapes which are called *bi-monotone* patches. *Monotone* is a concept defined on a regular mesh of quadrilaterals. If the number of quadrilateral segments (i.e., continuous sequences of quadrilaterals) is one for every column or row, the mesh is called *monotone*. A mesh that is monotone in both the column and the row directions is referred to as bi-monotone (see Fig. 1(b, c)).

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Our target is to produce good segmentation for fitting surfaces in the post processing step. In practice, the data points are preferred to have as uniform gaps as possible to each other so as to generate a better surface. In a bi-monotone patch, there are no holes or large inward curved regions and data points corresponding to an iso-parametric curve are always in one segment. In other words, there is no big gap in the intervals of the data points. Therefore, it is expected to be able to determine a good parameterization of the data points in the bi-monotone patch.

This study aims to generate large-size bi-monotone patches where the feature curves reside on the boundary. We first outline the partitioning a quadrilateral mesh into four-sided patches, then describe the detection of feature curves inside these patches. If a patch has feature curves, we split it into bi-monotone patches along these curves. Finally, patch configuration is improved to generate large patches. As it is undesirable to obtain patches whose shapes are far away from quadrilaterals, we control the shape using a *shape control* parameter that can be adjusted by the user during the patch generation process. In this study, we mainly target shape models of mechanical parts which consist of major smooth surfaces with feature curves between them.

1.1. Related work

Quad remeshing Many studies have explored methods of quad remeshing and a number of significant surveys in the field have been performed. A brief review of these techniques is given here [1,2]. Some methods [3,4] propose an anisotropic remeshing algorithm which explicitly trace curves along the principal curvature directions. Such approaches generate quad-dominant meshes with many extraordinary vertices. Some other methods [5–7] have involved the initial construction of a globally smooth parameterization of the mesh and the subsequent extraction of a quadrilateral mesh as a collection of iso-parameter lines. [7] reduces the quadrangulation problem into mixed-integer problems and converts the given triangular mesh into a quadrilateral mesh by optimizing quality aspects such as element quality, orientation, alignment and global structure (i.e., the distribution of extraordinary vertices). Some other recent methods [8–11] can also generate quadrilateral meshes of good quality.

Mesh segmentation Mesh segmentation techniques have a long tradition in the graphics community and a considerable body of literature surveys [12,13] on the topic exists. Some methods [14–16] involve utilizing surface curvature characteristics to segment a mesh into sub-meshes. [15] applies morphological watersheds (an image segmentation technique) to 3D surfaces, and [16] partitions the mesh while iterating between region growing and surface fitting. Other methods [17–20] involve using a geometric measure or filter for clustering or approximating mesh elements. [17] proposes a hierarchical clustering methodology utilizing edge contraction on a dual graph of the mesh, while [18] separates the mesh into sub-meshes, each approximated by geometrical primitives. [20] extends the optimization technique of geometric surface approximation [19] by allowing spheres, cylinders and rolling-ball blend patches in addition to planes.

Recently a number of mesh partitioning methods [21–23] involving decomposition of a given mesh into compact quad sub-meshes have been proposed. [21] suggests a method based on the motorcycle graph of Eppstein and Erickson [24] to partition semi-regular quadrilateral meshes into structured quadrilateral meshes. [23] optimizes the global structure of a given quadrilateral mesh by detecting and removing helices without changing the number and the position of singularities. The base complex (the union of all parametric lines starting and ending at singularities) of the optimized quadrilateral mesh is a coarse quadrilateral mesh, and is therefore good for fitting with surface patches. [22] proposes an algorithm to generate coarse quadrilateral patches which are appropriate to fit with T-splines. Here, a greedy optimization approach is applied to find a solution to several trade-offs in consideration of quality aspects such as patch quality, approximation, mesh complexity, orientation and alignment.

2. Method overview

In this section, we describe the flow of the proposed method and define some fundamental elements used in the algorithm.

2.1. Algorithm flow

Fig. 2 illustrates the main steps of the proposed algorithm. We adopt a quadrilateral mesh generated using a mixed integer quadrangulation algorithm of Bommes et al. [7] to generate large-size bi-monotone patches. Section 3 describes an algorithm which partitions the mesh into quadrilateral patches. Then a feature detection algorithm will be explained in Section 4. Sections 5 and 6 detail the generation of the bi-monotone patches.

2.2. Quadrilateral mesh

A quadrilateral mesh is a polygonal mesh in which all faces are quadrilaterals, and can be defined by a set of vertices embedded in R^3, a set of edges and a set of quadrilateral faces. A vertex in a quadrilateral mesh is called *ordinary* if its valence is four for interior vertices, or two or three for boundary vertices. Vertices that are not ordinary are called *extraordinary*.

Parametric directions starting from singularities can be defined topologically at each ordinary vertex of a quadrilateral mesh as $u, v, -u, -v$ by counter-clockwise labeling. Furthermore, tracing edges based on these parametric directions on ordinary vertices generates parametric lines. As a result, the quadrilateral mesh can be used for surface parameterization, and therefore for generating parametric surfaces.

3. Extended motorcycle graph algorithm

The first part of the proposed mesh decomposition involves tracing the edges of a given quadrilateral mesh using the motorcycle graph algorithm proposed by [21]. The mesh is then partitioned into several sub-meshes with boundaries formed by the motorcycle graph edges. In this step, the extraordinary vertices (valence $\neq 4$) are removed and located at the corners of these sub-meshes.

The motorcycle graph algorithm decomposes a quadrilateral mesh into several sub-meshes without extraordinary vertices; these sub-meshes are referred to as *structured meshes*. However, the resulting sub-meshes are not necessarily flat; they may have non-flat faces because the motorcycle graph algorithm does not take mesh geometry into account. In this paper, we propose an extended motorcycle graph (EMG) algorithm and promote the generation of sub-meshes with fewer non-flat faces.

The aim of the study outline in [21] is to decompose semi-regular quadrilateral meshes into structured quadrilateral sub-meshes based on the motorcycle graph concept of [24]. To achieve this, particles are placed on each extraordinary vertex and moved outward from the extraordinary vertices along one edge in each time step. When a particle reaches an ordinary vertex, it continues to move along its opposite edge. If a particle meets a boundary or a previously traversed vertex, it stops. When two particles meet at a vertex perpendicularly, the *right-hand rule* is used and one stops, while the other keeps going. Moreover, if three or four particles meet at a vertex simultaneously, they all stop.