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Adaptive patch-based mesh fitting for reverse engineering

Hongwei Lin*, Wei Chen, Hujun Bao

State Key Lab of CAD&CG, Zhejiang University, Hangzhou 310027, China

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Abstract

In this paper, we propose a novel adaptive mesh fitting algorithm that fits a triangular model with G^1 smoothly stitching bi-quintic Bézier patches. Our algorithm first segments the input mesh into a set of quadrilateral patches, whose boundaries form a quadrangle mesh. For each boundary of each quadrilateral patch, we construct a normal curve and a boundary-fitting curve, which fit the normal and position of its boundary vertices respectively. By interpolating the normal and boundary-fitting curves of each quadrilateral patch with a Bézier patch, an initial G^1 smoothly stitching Bézier patches is generated. We perform this patch-based fitting scheme in an adaptive fashion by recursively subdividing the underlying quadrilateral into four sub-patches. The experimental results show that our algorithm achieves precision-ensured Bézier patches with G^1 continuity and meets the requirements of reverse engineering.

Keywords: Reverse engineering; Adaptive fitting; Fitting error; G^1 continuity; Bézier patch; Triangular mesh

1. Introduction

In reverse engineering, the reconstructed triangular mesh from a point cloud contains many vertices and needs to be fitted with a smooth surface, e.g., smooth parametric patches. The triangular mesh is first segmented into a sequence of quadrilateral patches (see Fig. 1(a)), each of which is a triangular mesh with four boundaries. A set of Bézier (B-spline) patches are then built to fit the mesh vertices of these quadrilateral patches with two requirements, i.e., the fitting error of each patch and the G^1 continuity between each pair of neighboring patches. Satisfying these two requirements simultaneously can be achieved with an adaptive fitting scheme. That is, if the fitting error of one Bézier (B-spline) patch is beyond a given tolerance, its corresponding quadrilateral patch must be subdivided into four patches, which are further fitted with four smoothly stitched Bézier (B-spline) patches. This procedure proceeds until the fitting error of each Bézier (B-spline) patch reaches a given tolerance.

The continuous stitching problem among neighboring patches arises in interpolating a quadrangle curve mesh (see Fig. 1(b)) with parametric patches. Normally, the quadrangle curve mesh is manually generated by users, for instance, during designing a car body or ship hull. It consists of a set of smooth curves, and has fewer vertices than the reconstructed triangular mesh in reverse engineering. Here, the Bézier (B-spline) patches that interpolate these smooth curves are only required to be G^1 continuously stitched. Adaptive fitting is unnecessary in this case. Therefore, we will present the related work in Section 1.1 mainly on the continuity problem.

In fact, there has been much work which cares about the fitting error, but results in surfaces with weak continuity. For instance, Milroy et al. [1] proposed a B-spline surface fitting approach that leads to an uncomfortable visual appearance due to the lack of smoothness. Eck and Hoppe [2] developed an automatic method for fitting irregular meshes using bi-cubic Bézier patches. Its resulting surface has ε–G^1 continuity. Although the algorithm proposed by Krishnamurthy and Levoy [3] can fit B-spline surfaces with arbitrary topology, there is little discussion on the continuity of the resulting B-spline surfaces. Shi and Wang [4] introduced a local scheme for constructing convergent G^1 (not true G^1) smooth bi-cubic B-spline patches with interior single knots over a given arbitrary quadrangular partition of a polygonal model. Note that all these methods are incapable of adaptive fitting, and their fitting error cannot be guaranteed. However, in reverse engineering,

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* Corresponding author. Tel.: +86 571 88206681x518; fax: +86 571 88206680.

E-mail address: hwlin@cad.zju.edu.cn (H. Lin).

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precision-ensured fitting is important to the downstream CAD applications.

Till now, all work with adaptive fitting capability either makes use of triangular Bézier–Bernstein patches [5,6], or employs T-spline surfaces [7]. Whereas the quadrilateral Bézier patch is more preferred because it satisfies the NURBS standard, and is more popular than the triangular patch and T-spline in reverse engineering.

In this paper, a new *patch-by-patch* scheme is proposed to construct G^1 continuously stitching Bézier patches for the purpose of adaptively fitting the mesh vertices of the quadrilateral patches with precision-ensured results. We compute the normal vectors at the mesh vertices of the triangular mesh by averaging the normal vectors of their adjacent triangular patches. Along each boundary of each quadrilateral patch, the normal vectors form a fence which encloses and separates each quadrilateral patch (see Fig. 1(c)). This is different from the quadrangle curve mesh (Fig. 1(b)). We also generate a *normal curve* for each boundary by fitting the normal vectors with a quadratic Bézier curve. The normal curves make it possible to construct G^1 continuously stitching Bézier patches in a patchwise way. The key to the feasibility of the *patch-by-patch* scheme lies in that each fitting Bézier patch interpolates the normal curves on its four boundaries. Thus, adjacent Bézier patches share the same normal vectors on the common boundary, and they are tangent plane continuous (that is, G^1 continuous) along the boundary [8].

More importantly, the *patch-by-patch* scheme makes the adaptive fitting feasible, because the G^1 continuity of the patch stitching on the T-conjunctions which are caused by recursive subdivision, can be achieved by interpolating the normal curve on each boundary of every sub-quadrilateral patch. Surely, the fitting error is improved in an adaptive fashion.

The rest of this paper is organized as follows. In Section 1.1, we briefly review the related work. The overview of our approach is presented in Section 2. In Section 3, we introduce how to construct the normal curve mesh and boundary-fitting curve mesh. We present our *patch-by-patch* scheme, and show how to construct initial Bézier patches over the quadrilateral patches in Section 4. The adaptive fitting approach is described in Section 5. The experimental results are given in Section 6. Finally, we conclude the whole paper in Section 7.

1.1. Related work

Here, we briefly review the related work on interpolating a quadrangle curve mesh using G^1 smoothly stitching parametric patches. In general, these methods pay more attention on continuously stitching, and neglect the capability of ensuring the fitting error. They can be roughly classified into two categories according to the patch type: Bézier patch and B-spline patch.

One pioneering work using Bézier patch was proposed by van Wijk [9] for generating a smooth surface over a non-rectangular mesh with bi-cubic patches. Shirman and Séquin [10,11] employed five bi-cubic patches to interpolate each quadrilateral in a mesh of cubic curves. Peters [12] proposed a local ε–G^1 continuity scheme, to construct a smooth spline surface using bi-quadratic and bi-cubic Bézier patches. Later, Peters [13] constructed a G^1 smooth bi-quartic Bézier surface over a refined network of quadrilateral sub-cells generated by the midpoint mesh refinement technique. By subdividing each Bézier patch into nine small patches, Ma and Peng [14] obtained a G^1 smooth surface. Reif [15] generated G^1 smooth surfaces using bi-quadratic rectangular Bézier patches over semi-regular meshes. Ye and Nowacki [16] employed G^1 smooth bi-quintic Bézier patches to interpolate rectangular cubic curve meshes. Ye [17] also extended this method for constructing G^2 Bézier surfaces by interpolating a given G^2 quintic curve meshes.

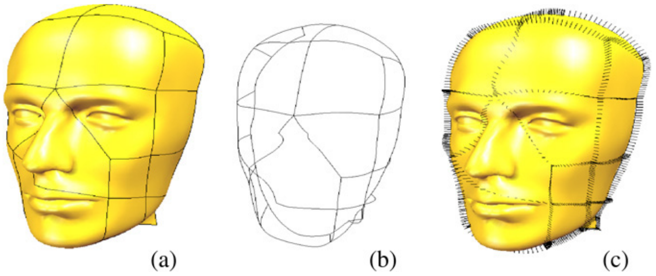

Fig. 1. (a) A triangular mesh is segmented into a set of quadrilateral patches. (b) The quadrangle curve mesh formed by the boundaries of the quadrilateral patches. (c) The normal curves enclose and separate all quadrilateral patches like a fence.

In terms of B-spline patches, Peters [18] constructed G^1 smooth bi-cubic B-spline patches with interior double knots generated by Catmull-Clark subdivision. Shi and Wang [19] developed a local processing scheme for constructing G^1 smooth bi-cubic B-spline surfaces with at least two pairs of interior double knots. Further, Shi et al. [20] improved this method to construct G^1 smooth bi-quartic B-spline patches with one pair of interior double knots. More recently, Shi et al. [21] proposed to constructs G^1 smooth B-spline surfaces with single interior knots over arbitrary topology. Another work by Kruth and Kerstens [22] incorporates positional, tangential or curvature continuity conditions with non-uniform rational B-splines in the CAD modeling of free-form surfaces.

2. Overview

Suppose that there is an oriented triangular mesh M. The normal vector at each vertex is computed by normalizing the average of the normal vectors of its adjacent triangular faces. The triangular mesh M is segmented into a set of quadrilateral patches, which are triangular mesh patches with four boundaries (Fig. 2(a)). The boundaries of all patches compose a quadrangle curve mesh Q (see Fig. 1(b)).

The adaptive mesh fitting approach includes the following steps:

1. Fit the normal vectors at the mesh vertices on each boundary of the quadrangle curve mesh Q with a quadratic Bézier curve, called a *normal curve* (see Fig. 2(b)).
2. Fit the mesh vertices on each boundary of the quadrangle curve mesh Q with a quintic Bézier curve, called a