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# A complementarity model for solving stochastic natural gas market equilibria

Jifang Zhuang, Steven A. Gabriel *

*Project Management Program, Department of Civil and Environmental Engineering, University of Maryland, College Park, Maryland 20742, USA*

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**Abstract**

This paper presents a stochastic equilibrium model for deregulated natural gas markets. Each market participant (pipeline operators, producers, etc.) solves a stochastic optimization problem whose optimality conditions, when combined with market-clearing conditions give rise to a certain mixed complementarity problem (MiCP). The stochastic aspects are depicted by a recourse problem for each player in which the first-stage decisions relate to long-term contracts and the second-stage decisions relate to spot market activities for three seasons. Besides showing that such a market model is an instance of a MiCP, we provide theoretical results concerning long-term and spot market prices and solve the resulting MiCP for a small yet representative market. We also note an interesting observation for the value of the stochastic solution for non-optimization problems.


*Keywords:* Natural gas; Complementarity; Stochastic; Market equilibrium

## 1. Introduction

Market equilibrium models for the natural gas markets abound in the literature both for the North American and European markets as described in Egging and Gabriel (2006) and Gabriel et al. (2005a). Despite the abundance of such works, incorporating uncertainty endogenously into these models is relatively unstudied. This stochastic nature arises from uncertain weather, market forces, regulations as well as general economic and engineering considerations. In addition, the reliability of the system can be challenged by terrorist attacks. Enhanced security thus provides

* Corresponding author.
*E-mail addresses:* zhuang@umd.edu (J. Zhuang), sgabriel@umd.edu (S.A. Gabriel).


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the motivation for how to keep the system running at minimal cost and greatest reliability given such uncertain events.

Stochastic programming is a framework for modeling optimization problems that involve uncertainty, which has been extensively used in energy models over the years. However, in contrast to the large volume of models relating to electricity investments and operations, e.g., Hobbs (1995) and Murphy et al. (1982b), the application of stochastic optimization equilibrium models to the oil and gas markets is more limited (Wallace and Fleten, 2003). Moreover, the field of algorithms for solving stochastic nonlinear complementarity problem/variational inequalities (NCP/VIs) is still relatively limited. Exceptions are Gurkan et al. (1999) who used a sample-path method, a simulation-based scheme to solve stochastic variational inequalities and Belknap et al. (2000) who analyzed the stochastic NCP/VIs using a quasi-Monte-Carlo simulation technique and relied on the gradient information at sample solution points.

In addition, more recently, Cabero et al. (2005) has proposed an approach for solving stochastic complementarity problems in electricity using a Benders Decomposition method and Shanbhag et al. (2005) has proposed a decomposition-based method for a stochastic complementarity problem involving oligopolistic players in spot and forward markets in electricity.

The natural gas model in this paper is an instance of a mixed complementarity problem (MiCP), derived from the Karush–Kuhn–Tucker (KKT) conditions of the stochastic optimization problems faced by various participants and a series of market-clearing conditions. Previous similar work includes: a stochastic dynamic Nash–Cournot model of imperfect competition for studying the contracts in the European gas market proposed by Haurie et al. (1987); a Stackelberg–Nash–Cournot equilibrium model with a numerical illustration of the European gas market by De Wolf and Smeers (1997). In contrast to these previous efforts, the current paper provides substantially more details on the market players and proves relevant theorems about equilibrium values — these are two of the main contributions in addition to implementation on a sample network.

The stochastic equilibrium model presented in this paper, denoted S-NGEM, for stochastic natural gas equilibrium model, is an extension of the deterministic market equilibrium model proposed in Gabriel et al. (2005a,b), abbreviated as NGMEP (Natural Gas Market Equilibrium Problem). S-NGEM aids market participants in planning the sales or purchases under uncertainty and considers both a long-term and spot market for gas, where consumer demand over the time horizon is subject to probabilistic distributions. Whereas, the long-term market provides planning level contracts (with reservation charges) which must be made at the beginning of the time horizon, the spot market contracts are available for delivery at the beginning of each season. All activities that take place in the spot market are merely committed to a season. Participants have the flexibility to make adjustment to their activity levels every season in response to the random outcomes. In an extreme case when the number of long-term trading periods tends to infinity, the market equilibrium tends to the perfect competition solution (Allaz and Vila, 1993).

In addition to the previously mentioned contributions, the new work explores, following Birge and Louveaux (1997), the value of the stochastic solution (VSS) and the expected value of perfect information (EVPI) are re-defined for the case of a stochastic equilibrium model. Interestingly, a negative VSS and EVPI in the case study were observed unlike the case for stochastic optimization problems. These negative values were checked on a smaller market equilibrium problem to make sure numerical inaccuracies were not the reason for the negative values.

Being an attempt in a relatively unstudied area, this work has captured market features that have not been studied by previous works. On the other hand, due to the complication of the reality and limitation of the parallel studies, the model has made simplifications of the market. A major one is that there is no switching possibility for all market participants between the long-term and