J. Math. Anal. Appl. 382 (2011) 231–247

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Lipschitz continuity and duality for dynamic oligopolistic market equilibrium problem with memory term

Annamaria Barbagallo [a,*], Rosalba Di Vincenzo [b]

[a] Department of Mathematics and Applications "R. Caccioppoli", University of Naples "Federico II", via Cintia, 80126 Naples, Italy
[b] University of Catania, Department of Mathematics and Computer Science, viale Andrea Doria n. 6, 95125 Catania, Italy

ARTICLE INFO

Article history:
Received 9 March 2010
Available online 20 April 2011
Submitted by A. Daniilidis

Keywords:
Oligopolistic market equilibrium problem
Memory term
Lipschitz continuity
Lagrange duality

ABSTRACT

We are concerned with an infinite dimensional variational inequality which is connected with the dynamic oligopolistic market equilibrium problem. We will provide existence theorems and show, under minimal assumptions on the data, the Lipschitz continuity of the solution. Moreover a general duality theory is provided overcoming the difficulty of the voidness of the interior of the ordering cone which defines the cone constraints.

1. Introduction

The aim of the paper is to study from several points of view the following variational inequality.

Find $x^* \in \mathbb{K}$ such that

$$\int_0^T \sum_{i=1}^m \left\langle -\frac{\partial v_i(t, x^*(t))}{\partial x_i} + \widetilde{l}_i(t), x_i(t) - x_i^*(t) \right\rangle dt \geqslant 0, \quad \forall x \in \mathbb{K}, \tag{1}$$

where $\frac{\partial v_i}{\partial x_i}$ and $\widetilde{l}_i$ are vector-functions belonging to $L^2([0,T],\mathbb{R}^n)$ and

$$\mathbb{K} = \left\{ x \in L^2\left([0,T],\mathbb{R}^{mn}\right) \colon 0 \leqslant \underline{x}(t) \leqslant x(t) \leqslant \bar{x}(t), \text{ a.e. in } [0,T] \right\} \tag{2}$$

(see Sections 2 and 3 for notations and details).

Then we are dealing with an infinite dimensional variational inequality which, as we will show in the next sections, is connected with the dynamic oligopolistic market equilibrium problem. This problem due to Cournot in [15], recently, has been studied in the dynamic case (see [7]). Taking into account of the recent paper [34], we will provide general existence theorems for (1). Then, using only Lipschitz continuity assumptions on the data, we will prove that the solution to (1) is Lipschitz continuous with respect to the time $t \in [0,T]$ and, to this aim, we have to estimate the variation rate of projections onto time-dependent constraints set. Further we provide a study of a Lagrange theory applied to (1). To this regard we remark that the usual constraint qualification conditions fail in our case, because the interior of the ordering cone which defines the cone constraints is empty. Then we are forced to apply a new infinite dimensional duality theory (see [20,17,35]) which does not require this kind of assumptions and which is very effective, rather than more famous

* Corresponding author.
E-mail address: annamaria.barbagallo@unina.it (A. Barbagallo).

0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2011.04.042

theories (as the Liusternik, Tikhomirov and Goldstine ones). In such a way we can find the dual variables which lead to a simple description of important features of the solution to (1). We recall that the existence of Lagrange multipliers has been obtained for other equilibrium problems, as dynamic equilibrium problems (see [22,23,7]) and the dynamic Walrasian problem (see [26,24]), in [17,18,29,9,25]. Moreover, it is worth remarking that A. Mas-Colell (see [32]) was one of the first who has dealt with the emptiness of the ordering cone in connection with the price equilibrium existence problem. A. Mas-Colell overcome the problem of the emptiness considering as setting a topological vector lattice for which the ordering cone is convex, monotone and uniformly proper and proving an existence theorem for such problem. However not too many examples of such spaces are given.

Now we would like to say some words in order to clarify the presence of the time in variational inequality (1). We can recall that M.J. Beckmann and J.P. Wallace in [10] were the first to point out the convenience to consider the evolution in the time of the equilibrium conditions. In fact they claim that "the time-dependent formulation of equilibrium problems allows one to explore the dynamics of adjustment processes in which a delay on time response is operating". Of course a delay on time response always happens because the processes have not an infinite speed. The possibility of adjustment processes can be obtained considering the evolution in the time of the cost function and, as a consequence, of the solution. Moreover, the adjustment processes of the cost is better specified considering a memory term which one can assume of the type of the Volterra operator if, as we made, it is supposed that the behavior of physical and economic models, in a deterministic framework, is the same. In (1) the influence of the adjustment processes is given by the function $(\widetilde{I}_i)_{i=1,\dots,m}$, which derives from the memory term acting in the profit function

$$\left(v_i\big(t, x(t)\big) - \int_0^t \sum_{k=1}^n I_{ik}(t-s)x_{ik}\,ds\right)_{i=1,2,\dots,m} \tag{3}$$

(see Section 3), where $\widetilde{I}_i(t) = (\int_0^t I_{ij}(t-s)\,ds)_{j=1,\dots,n}$, a.e. in $[0,T]$, for $i=1,\dots,m$.

It is also worth mentioning that the solutions to (1) are the critical points of an associated projected dynamical system and, then, there exists a strict connection between evolutionary variational inequalities and projected dynamical systems (see [12–14,19,28]).

The rest of the paper is structured as follows. In Section 2, we introduce an evolutionary variational inequality that expresses some equilibrium problems, in particular, a basic model of an oligopolistic market equilibrium problem when an evolution in time occurs (see [7]). In Section 3, we study evolutionary variational inequality (1) that expresses the dynamic oligopolistic market equilibrium problem in presence of delay by a long-term memory. Section 4 is devoted to show some existence results for solutions to variational inequality (1). In Section 5, a Lipschitz continuity result is showed. In Section 6, we generalize to our case the Lagrange multipliers theorem proved for the dynamic oligopolistic market equilibrium problem in [9]. At last, in Section 7, an example concludes the paper.

2. Dynamic oligopolistic market equilibrium

In order to present evolutionary variational inequality (1), first of all, we consider a variational inequality without the memory term $(\widetilde{I}_i)_{i=1,\dots,m}$, namely

$$\int_0^T \sum_{i=1}^m \left\langle -\frac{\partial v_i(t, x^*(t))}{\partial x_i}, x_i(t) - x_i^*(t)\right\rangle dt \geqslant 0, \quad \forall x \in \mathbb{K}, \tag{4}$$

where $\mathbb{K}$ is given by (2).

Variational inequality (4) expresses some dynamic equilibrium problems in which the equilibrium conditions are assigned maximizing an operator. In particular, the dynamic oligopolistic market conditions are characterized by (4), where the functions v_i, x_i and x_i^*, for $i=1,\dots,m$, represent some precise elements of the model.

Let us introduce, for the reader's convenience, the dynamic oligopolistic market equilibrium problem. This model constitutes an example of imperfect competition and can be viewed as a prototypical game theoretic problem, operating under the Nash equilibrium concept of noncooperative behavior.

Let us consider m firms P_i, $i=1,2,\dots,m$, and n demand markets Q_j, $j=1,2,\dots,n$, that are generally spatially separated. Assume that the homogeneous commodity, produced by the m firms and consumed at the n markets, is involved during a period of time $[0,T]$, $T>0$. Let $p_i(t)$, $t\in[0,T]$, $i=1,2,\dots,m$, denote the nonnegative commodity output produced by firm P_i at the time $t\in[0,T]$ and let $q_j(t)$, $t\in[0,T]$, $j=1,2,\dots,n$, denote the demand for the commodity at demand market Q_j at the time $t\in[0,T]$. Let $x_{ij}(t)$, $t\in[0,T]$, $i=1,2,\dots,m$, $j=1,2,\dots,n$, denote the nonnegative commodity shipment between the supply market P_i and the demand market Q_j at the time $t\in[0,T]$. We group the production outputs into a vector-function $p:[0,T]\to\mathbb{R}^m_+$, the demands into a vector-function $q:[0,T]\to\mathbb{R}^n_+$, and the commodity shipments into a matrix-function $x:[0,T]\to\mathbb{R}^{mn}_+$.

Assuming that we are not in presence of production and demand excesses the following feasibility conditions must hold for every i and j and a.e. in $[0,T]$: