Does the return-state-varying relationship between risk and return matter in modeling the time series process of stock return?

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ABSTRACT

This paper empirically investigates the time series behavior of stock returns and volatility and the relationship between return premium and stock market risk by utilizing a TSV-GARCH(p,q)-Risk-Mean model. The empirical findings of this paper provide evidence for the distinct driving forces in mean and volatility and the state-dependent tradeoff between risk and return. The empirical results demonstrate that the stock market displays four types of dynamic processes (high-return–low-volatility state, low-return–low-volatility state, high-return–high-volatility state, and low-return–high-volatility state) and that the structural change process of stock market returns is greatly at odds with that of stock market volatility. The TSV-GARCH(1,1)-Risk-Mean model provides better in-sample fit compared to the conventional GARCH(p,q) and regime-switching GARCH(p,q) models. Moreover, the relationship between excess returns and risk is positive, and the intensity of this positive relationship during periods of bear market is significantly higher than that during periods of bull market, which provides supporting evidence for the countercyclical risk premiums hypothesis in which the magnitude of compensation for enduring risk is weaker during periods of favorable financial conditions than during periods of adverse conditions.

1. Introduction

Investment in the stock market is one of the most popular investment tools in the world, and stock return processes have been discussed in numerous studies over several decades. To date, modeling stock return dynamics is a hot topic and attracts plenty of attention in empirical research. In addition to outstanding properties, such as an autoregressive process in the first two conditional moments and asymmetric density with a skewed shape, recognized by the overwhelming majority of researchers, structural changes in mean and/or variance, and risk–return relationships have gained a lot of attention in recent years. How to detect structural changes and address the relationship between excess return and risk is a non-trivial task. For these reasons, this paper intends to investigate whether an empirical model that considers more flexible Markov-switching specifications and which takes distinct Markov-switching processes in mean and variance as well as possible return-state-varying relationships between return and risk into account, can enhance the fit performance of stock returns.

Regime-switching models that detect structural breaks of macroeconomic and financial data endogenously through an unobserved state variable with a first-order homogenous Markov chain have received widespread attention and approval. A variety of

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Markov-switching models, which simultaneously govern the switching characteristics of return and volatility, have been extensively applied in empirical studies.[1] The main assumption behind these studies is that the switching behavior in the first two moments relies on the same state variable, which follows a discrete-time Markov process. Explicitly, the structural change dates and probabilities of regime switching are the same for both the dynamics of return and volatility.

However, empirical studies, such as McConnell and Perez-Quiros (2000); Lettau, Ludvigson, and Wachter (2008); and Liow, Chen, and Jingran (2011), demonstrate that the restriction, which assumes that the processes of mean and variance are governed by the same state variable, is inappropriate for financial and macroeconomic data. In analyzing the securitized real estate market, Liow et al. (2011) find statistically significant evidence to show that the structural break points in the return series are different from those in the volatility series. McConnell and Perez-Quiros (2000) and Lettau et al. (2008) employ a simplified Markov specification, which assumes two independent state variables, to investigate output growth rate and consumption growth rate, respectively, and to show that the dynamics of the first two moments differ significantly. Hence, the magnitude of improvement in fit performance when two state variables are included in the stock return dynamics is worth further investigation.

A number of empirical studies have tried to determine the possible relationship between risk and return, including time-invariant and time-variant forms. The time-variant specification takes into account changes in the investment environment,[2] while the time-invariant form does not. Empirical studies observe that both positive and negative relationships are possible outcomes.[3] However, Pastor and Stambaugh (2001) provide preliminary evidence relating to the view that taking into account the possible relationship between equity premium and volatility can help one to identify structural changes in stock return. Hence, whether stock return dynamics can be more adequately modeled if the time-variant relationship between excess return and risk is incorporated into the econometric specification is another important concern.

The primary aim of this empirical study is to investigate whether the number of state variable, the time-variant trade-off between risk and return, and the order of ARCH (autoregressive conditional heteroskedasticity) and GARCH (generalized autoregressive conditional heteroskedasticity) terms are important factors in in-sample fitting ability. The empirical specification used in this paper is a Markov-switching GARCH(p,q) model with two state variables and a return-state-varying relationship between excess return and risk. This model is abbreviated as TSV-GARCH(p,q)-Risk-Mean model. The TSV-GARCH(p,q)-Risk-Mean model is not only a variation of the Markov-switching GARCH(1,1)-in-Mean model of Kim and Lee (2008), who examine the state-dependent risk premium, but also an extended version of Lettau et al. (2008), who propose a specification with two independent Markov-switching variables but without considering the GARCH(p,q) process. Briefly speaking, the empirical model is a four-state Markov-switching GARCH(p,q) model with a risk measure in mean. The advantages of the empirical model are presented below.

First, compared with existing empirical studies, which assume two market states (such as high-return/low-return states or high-volatility/low-volatility states), the model use in this paper includes two state variables. Following the specification of McConnell and Perez-Quiros (2000) and Lettau et al. (2008), one state variable captures two distinct return processes (a high-return state and low-return state) and the other state variable captures two different volatility processes (a high-volatility state and low-volatility state). Hence, the stock market dynamics can be grouped into four types: a high-return–low-volatility state, low-return–low-volatility state, high-return–high-volatility state, and low-return–high-volatility state. Given associations among trading volume, the absolute value of price changes and variance, the higher stock market volatility may result from two extreme stock market conditions: stock market crashes and booms.[4] Hence, high volatility may be accompanied by the stock price crashing or rising. It is clear from the above argument then that allowing the occurrence of high volatility to rely only on stock price surges is too simple to properly depict the dynamics of stock returns. Briefly speaking, the new specification can help investors to extract information on the direction of stock price movement based on conditional variance.

The second contribution is that the TSV-GARCH(p,q)-Risk-Mean model is able to investigate whether a risk–return relationship is positive or negative as well as whether the risk premium is procyclical or countercyclical. In contrast to the specification of Kim and Lee (2008), in this paper, the future risk measure, similar to the opinion of Turner, Startz, and Nelson (1989) and Kim, Morley, and Nelson (2004), is measured by the predicting probability of a high-volatility state instead of conditional variance. In order to investigate the procyclical or countercyclical perspectives, the coefficient of the risk premium is related to the state variable of the stock return. In this study, through the return-state-varying coefficients of the risk premium, the procyclical or countercyclical pattern can be identified. A countercyclical pattern exists if the positive risk–return relationship is stronger in the bear market than in the bull market, and by contrast, a procyclical pattern indicates that the positive coefficient of the risk premium is larger in the bull market than in the bear market.

The third advantage of this empirical model is that it is able to investigate whether an empirical model, which includes two Markov-switching state variables, a higher-order GARCH process and a state-varying risk–return relationship, can enhance fit performance. To the best of my knowledge, almost all studies that examine the risk–return relationship pay attention to the

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[1] See, for instance, Gray (1996); Sarno and Valente (2000); Ang and Bekaert (2002, 2004); Fong and Kim (2002, 2003); Marcucci (2005); Chen (2007, 2009, 2010) and Chen and Tsay (2011).

[2] See, for example, Campbell and Cochrane (1999); Harvey (2001); Brandt and Kang (2004); Lettau and Ludvigson (2010); Mayfield (2004); Bali and Peng (2006); Li (2007); Lundblad (2007); Kim and Lee (2008) and Rossi and Timmermann (2010).

[3] Some studies, such as those by French, Schwert, and Stambaugh (1987); Engle, Lilien, and Robins (1987); Bollerslev, Engle, and Wooldridge (1988); Turner et al. (1989); Campbell and Hentschel (1992); Bansal and Yaron (2004); Kim et al. (2004); Lettau and Ludvigson (2010); Ghysels, Santa-Clara, and Valkanov (2005); Bali and Peng (2006); Lundblad (2007); Bali (2008); Bandi and Perron (2008) and Bali and Engle (2010), find a positive relationship between risk and return; in contrast, the studies of Nelson (1991); Glosten et al. (1993) and Whitelaw (1994, 2000) support a negative relationship.

[4] The opinion that high volatility may occur in boom and bust periods is verified by considering the positive relationship between trading volume and the absolute value of price change (Ying, 1966) along with the positive relationship between trading volume and volatility (Clark, 1973; Copeland, 1976).