Probabilistic Load Flow: A Review

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***Abstract*--This paper reviews the development of the probabilistic load flow (PLF) techniques. Applications of the PLF techniques in different areas of power system steady-state analysis are also discussed. The purpose of the review is to identify different available PLF techniques and their corresponding suitable applications so that a relatively accurate and efficient PLF algorithm can be determined for the concerned system, e.g. a distribution system with large integration of renewable energy based dispersed generations.**

***Index Terms*--Analytical approach, Monte Carlo, Probabilistic load flow, Stochastic load flow**

I. NOMENCLATURE

BSR : Bulk system reliability
CHP : Combined heat and power
CDF : Cumulative distribution function
CRE : Composite reliability evaluation
DG : Dispersed generation
DLF : Deterministic load flow
LF : Load flow
MC : Monte Carlo
PDF : Probabilistic density function
PLF : Probabilistic load flow
SLF : Stochastic load flow
WT: : Wind turbine

II. INTRODUCTION

THE DLF is used to analyze and assess the planning and operating of power systems on a daily routine. DLF uses specific values of power generations and load demands of a selected network configuration to calculate system states and power flows. Therefore, DLF ignores uncertainties in the power systems, e.g. the outage rate of generators, the change of network configurations and the variation of load demands. Furthermore, modern power systems with integration of DG units, such as WTs and photovoltaic systems, introduce additional power fluctuations into the system due to their uncontrollable prime sources. Therefore, the deterministic approach is not sufficient for the analysis of modern power systems and the results from DLF may give an unrealistic assessment of the system performance. In order to take the uncertainties into consideration, different mathematical approaches for uncertainty analysis can be used, such as the probabilistic approach, fuzzy sets and interval analysis [1]. The probabilistic approach has a solid mathematical background and has been applied to power systems in different areas [2]. This paper provides a review on the PLF techniques, which are used to analyze the system steady state performance.

The PLF was first proposed in 1974 and has been further developed and applied into power system normal operation, short-term/long-term planning as well as other areas[3][4][5]. The PLF requires inputs with PDF or CDF to obtain system states and power flows in terms of PDF or CDF, so that the system uncertainties can be included and reflected in the outcome. The PLF can be solved numerically, i.e. using a MC method, or analytically, e.g. using a convolution method, or a combination of them [6][7][8]. The main concern about the MC method is the need of large number of simulations, which is very time-consuming; whereas the main concerns about the analytical approach are the complicated mathematical computation and the accuracy due to different approximations. In parallel, a similar technique called SLF has also been developed to deal with the same problems [2]. It is based on the assumption that the probabilistic distributions of the system states and power flow outputs are normal distributions. This assumption, although it simplifies the calculation, is demonstrated to be unreliable by other researches [4]. Therefore, the application of the SLF is very limited and will not be further discussed in this paper.

Reference [9] provides an extensive bibliography on PLF published before 1988. Reference [2] also summarizes main techniques of PLF published before 1987. The main focuses of these literatures are on the linearization of LF equations, network outages and the interdependence among nodal power injections. However, there are also numerous literatures on PLF published from 1989 up to now, regarding issues such as the efficiency of algorithms, power system planning and the inclusion of voltage control devices [10][5][11]. A review on the traditional and newly developed PLF algorithms will provide a clearer indication on the different available techniques and corresponding application areas. A suitable PLF technique can thereafter be selected to cope with the concerned issues associated with modern power systems, such as distribution systems integrated with a large amount of stochastic DG. This paper is organized as follows. First, the basic techniques and related assumptions of the PLF technique are analyzed. Then miscellaneous techniques, developed to improve the accuracy and efficiency of the PLF algorithm, are discussed. Finally, the application and extension of the PLF technique in different areas of power systems are presented.

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III. Basic Techniques of PLF

The PLF can be performed by using either a numerical approach or an analytical approach. The numerical approach, e.g. a MC method, substitutes a chosen number of values for the stochastic variables and parameters of the system model and performs a deterministic analysis for each value so that the same number of values are obtained in the results; whereas the analytical approach analyzes a system and its inputs using mathematical expressions, e.g. PDFs, and obtains results also in terms of mathematical expressions.

A. Numerical Approach

The numerical approach is to adopt a MC method for the PLF analysis. The two main features of MC simulation are random number generation and random sampling. Software such as MATLAB provides algorithms for pseudorandom number generation. Refer to [12] for different techniques of random sampling, e.g. simple random sampling, stratified random sampling, etc. Although sampling techniques can be rather sophisticated, the PLF using MC is in principle doing DLF for a large number of times with inputs of different combinations of nodal power values. Therefore, the exact non-linear form of LF equations as shown in (1)-(5) can be used in the PLF analysis.

$$P_i = U_i \sum_{k=1}^{n} U_k \left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) \quad (1)$$

$$Q_i = U_i \sum_{k=1}^{n} U_k \left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) \quad (2)$$

$$P_{ik} = -t_{ik}G_{ik}U_i^2 + U_iU_k\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) \quad (3)$$

$$Q_{ik} = t_{ik}B_{ik}U_i^2 - B_{ik}^{'}U_i^2 + U_iU_k\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) \quad (4)$$

$$Q_{i(sh)} = U_i^2 B_{i(sh)} \quad (5)$$

where P_i and Q_i are the net active and reactive power injection at bus i; P_{ik} and Q_{ik} are the active and reactive power flows in line ik at the bus i side; U_i and U_k are the voltage magnitude at bus i and k; θ_{ik} is the angle difference between the voltages at bus i and k ; G_{ik} and B_{ik} are the real and imaginary part of the corresponding admittance matrix. The capability to use the exact non-linear LF equations is the reason why results obtained from the PLF using MC are usually taken as a reference to the results obtained from other PLF algorithms with simplified LF equations, so as to check the accuracy of the algorithms [13]. In spite of its relatively high accuracy, the MC method requires large amount of computation time due to the large number of LF calculations.

B. Analytical Approach

The basic idea of the analytical approach is to do arithmetic, i.e. using convolution techniques, with PDFs of stochastic variables of power inputs so that PDFs of stochastic variables of system states and line flows can be obtained. However, the difficulties of solving PLF equations by the convolution of PDFs of input power variables are mainly twofold [2][14]:

a) LF equations (1)-(5) are non-linear
b) input power variables at different buses are usually not completely independent of or linear-correlated

Therefore, a number of assumptions are usually made to be able to perform the PLF easily using an analytical approach. These assumptions are:

c) linearization of LF equations
d) total independent or linear-correlated power variables
e) normal distribution and discrete distribution are usually assumed for the load and generation, respectively
f) network configuration and parameter are constant

As a result, the LF equations are linearized around the estimated mean of the system states $\widehat{\mathbf{X}}$ with the first-order Taylor expansion [13]. If (1) and (2) are represented by a more general form as:

$$\mathbf{Y} = \mathrm{f}\left(\mathbf{X}\right) \quad (6)$$

then the linearized form can be expressed as:

$$\mathbf{X} \approx \widehat{\mathbf{X}} + \mathbf{A}\left(\mathbf{Y} - \overline{\mathbf{Y}}\right) \quad (7)$$

where

$$\mathbf{A} = \left(\left.\frac{\partial \mathrm{f}}{\partial \mathbf{X}}\right|_{\mathbf{X}=\widehat{\mathbf{X}}}\right)^{-1} \quad (8)$$

A is also referred to as sensitivity coefficient matrix in the PLF formulation. Similar expressions can be derived for (3), (4) and (5). In the DLF solved by using Newton-Raphson method, the Jacobian matrix **A** is also computed for each iteration until errors of the results are less than specified values. However, in the PLF here, the Jacobian matrix is only computed once for the computation of each LF. Therefore, errors caused by the linearization of LF equations should be noted and taken care of. Equation (7) shows that the system states are expressed by a linear combination of input power variables. With the assumption of independence, a convolution technique can then be applied to derive the PDFs of system states **X**, which is:

$$\mathrm{f}\left(\mathbf{X}_i\right) = \mathrm{f}\left(\mathbf{Y}_1 - \overline{\mathbf{Y}}_1\right) * \mathrm{f}\left(\mathbf{Y}_2 - \overline{\mathbf{Y}}_2\right) * \ldots * \mathrm{f}\left(\mathbf{Y}_n - \overline{\mathbf{Y}}_n\right) \quad (9)$$

Refer to [7][13][15] for detailed convolution techniques of mixed continuous and discrete variables.

IV. Improvement of PLF Techniques

A. Non-linear LF Equations

As the non-linear LF equations are linearized around the expected value region, the accuracy of the results become worse when values of the input power variables are far from their corresponding mean values. The errors are usually reflected in the tail regions of the results, e.g. the two ends of a distribution curve of a bus voltage. This may greatly impact the decision-making judged by adequacy indices such as the probability of a bus voltage outside its operational limits. Therefore, different methods have been proposed to mitigate the error caused by the linearization of the LF equations. Two typical solutions are PLF using multi-linearization [16][8] and the quadratic PLF [17][18].

Multilinearization of the PLF is to linearize the LF