ADAPTIVE FREQUENCY ESTIMATION OF THREE-PHASE POWER SYSTEMS WITH NOISY MEASUREMENTS

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ABSTRACT

We examine the problem of estimating the frequency of a three-phase power system in an adaptive and low-cost manner when the voltage readings are contaminated with observational error and noise. We assume a widely-linear predictive model for the $\alpha\beta$ complex signal of the system that is given by Clarke's transform. The system frequency is estimated using the parameters of this model. In order to estimate the model parameters while compensating for noise in both input and output of the model, we utilize the notions of total least-squares fitting and gradient-descent optimization. The outcome is an augmented gradient-descent total least-squares (AGDTLS) algorithm that has a computational complexity comparable to that of the complex least mean square (CLMS) and the augmented CLMS (ACLMS) algorithms. Simulation results demonstrate that the proposed algorithm provides significantly improved frequency estimation performance compared with CLMS and ACLMS when the measured voltages are noisy and especially in unbalanced systems.

Index Terms—adaptive frequency estimation, gradient-descent optimization, smart grids, total least-squares, widely-linear modeling.

1. INTRODUCTION

Smart grids collect and act on information regarding the behavior of the consumers and suppliers in an automated manner to enhance the efficiency, reliability, economy, and sustainability of the generation, distribution, and consumption of electrical energy [1].

System frequency is amongst the most important and sensitive parameters to be constantly monitored in the smart grids. Accurate power frequency estimation is crucial to check the health state of the power grid and assures reliable measurement of other system parameters such as voltages, currents, and active and reactive powers. Market economy will presumably drive power systems to operate much closer to their limits necessitating a perfect generation-load balance. Deviation of the system frequency from its rated value faithfully portrays an imbalance between the power generation and load demand. Accordingly, many power-system protection-and-control applications require accurate and fast estimation of the system frequency. An erroneous frequency estimate can result in insufficient load shedding by frequency relays, which in turn may ultimately cause a catastrophic grid failure [2].

Research on frequency estimation of power systems has been conducted for decades generating a copious body of literature (e.g., see [2]-[22] and references therein). Several methods have been proposed to estimate the power system frequency based on zero-crossing technique [4], phase-locked loop [5]-[8], least-squares adaptive filtering [9]-[11], and extended Kalman filter [12]-[14], to name a few. Most of these methods rely on the voltage readings of a single phase of the system. In three-phase systems, none of the single phases can necessarily characterize the whole system and its properties. Therefore, a robust frequency estimator should take into account the information of all three phases [15]-[18]. Applying Clark's transform (also known as $\alpha\beta$ transform), a single complex signal can be used to encompass the three-phase information [19]. It has been shown that the frequency of the three-phase power system can be estimated using a linear predictive model for this complex signal ($\alpha\beta$ signal) [20], [21]. However, since the $\alpha\beta$ signal is improper (its real and imaginary parts have different statistical properties) [22]-[24] when the system is unbalanced (e.g., phases feature different peak voltages), it is better described via a widely-linear model rather than a strictly-linear one [25], [26].

In [21], an algorithm for frequency estimation of three-phase power systems utilizing the $\alpha\beta$ signal is developed based on the widely-linear (augmented) complex least mean square (ACLMS) algorithm [27]. In unbalanced situations, this algorithm significantly outperforms its strictly-linear counterpart proposed in [20], which is based on the complex least mean square (CLMS) algorithm [28], while enjoying the simplicity and numerical stability of the LMS-type algorithms. However, it assumes a noise-free environment,

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i.e., where the voltage measurements are exact and error-free. Such assumptions are often unrealistic since several kinds of error can contaminate the measurements, e.g., sampling, quantization, and instrument errors. Therefore, in practice, this algorithm may have a poor estimation performance because of failing to account for the error in the signals.

Total least-squares (TLS) is a fitting method that improves the accuracy of the least-squares estimation techniques when both the input and output data of a linear system are subject to observational error. TLS minimizes the perturbation in the input and output data that is required to fit the input to the output [29]-[31].

In this paper, we develop a frequency estimation algorithm for three-phase power systems assuming noisy phase voltage observations. To this end, we utilize the concepts of TLS fitting and gradient-descent optimization to compute the parameters of a widely-linear predictive model considered for the $\alpha\beta$ signal. The system frequency is then calculated using the model parameters. Simulations testify that the performance of the new algorithm is superior to that of the ACLMS algorithm when the phase voltages are measured in noise while being almost as computationally efficient as ACLMS.

2. PROPOSED ALGORITHM

The voltages of a three-phase power system can be represented as

$$v_n^a = V_n^a \cos(2\pi f \tau n + \phi) + \eta_n^a$$

$$v_n^b = V_n^b \cos\left(2\pi f \tau n + \phi - \frac{2\pi}{3}\right) + \eta_n^b$$

$$v_n^c = V_n^c \cos\left(2\pi f \tau n + \phi + \frac{2\pi}{3}\right) + \eta_n^c$$

where V_n^a, V_n^b, and V_n^c are the peak values, f is the system frequency, τ is the sampling interval, n is the time index and ϕ is a constant phase while η_n^a, η_n^b, and η_n^c denote the observational errors and noises.

Using Clarke's ($\alpha\beta$) transform [19], i.e.,

$$\begin{bmatrix} v_n^\alpha \\ v_n^\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_n^a \\ v_n^b \\ v_n^c \end{bmatrix},$$

we obtain a complex-valued voltage signal as

$$v_n = v_n^\alpha + j v_n^\beta$$

that can be used for adaptive frequency estimation [32], [33]. Here, $j = \sqrt{-1}$.

A *widely-linear* predictive model for v_n is described as

$$\tilde{v}_{n-1} h + \tilde{v}_{n-1}^* g = \tilde{v}_n$$

or

$$[\tilde{v}_{n-1}, \tilde{v}_{n-1}^*] \begin{bmatrix} h \\ g \end{bmatrix} = \tilde{v}_n$$

where $\tilde{v}_n$ is the noiseless value of v_n, i.e., when $\eta_n^a = \eta_n^b = \eta_n^c = 0$, and superscript $*$ denotes complex-conjugate while h and g are the model parameters that we wish to identify. It is shown in [21] that, using h and g, the system frequency can be estimated as

$$\hat{f} \approx \frac{1}{2\pi\tau} \sin^{-1}\left(\sqrt{\Im^2(h) - |g|^2}\right)$$

where $\Im(\cdot)$ and $|\cdot|$ denote the imaginary part and the absolute value, respectively.

In order to identify h and g at the presence of noise, we utilize an adaptive filter whose tap weights vector, denoted by $\mathbf{w}_n = [w_{n,1}, w_{n,2}]^T$, is taken as an estimate for $[h, g]^T$ at iteration n. We wish to compute $\mathbf{w}_n$ such that it fits the filter input data to the desired filter output data by incurring minimum perturbation:

$$(\mathbf{X}_n^T + \boldsymbol{\Delta}_n)\mathbf{w}_n = \mathbf{y}_n + \boldsymbol{\delta}_n \tag{1}$$

where

$$\mathbf{X}_n = [\mathbf{x}_1, \dots, \mathbf{x}_{n-1}, \mathbf{x}_n],$$

$$\mathbf{y}_n = [v_1, \dots, v_{n-1}, v_n]^T,$$

$$\mathbf{x}_n = [v_{n-1}, v_{n-1}^*]^T,$$

superscript T stands for transpose, and $\boldsymbol{\Delta}_n \in \mathbb{C}^{n\times 2}$ and $\boldsymbol{\delta}_n \in \mathbb{C}^{n\times 1}$ denote the minimum input and output perturbations, respectively. Using the singular value decomposition (SVD) of the augmented data matrix, $[\mathbf{X}_n^T, \mathbf{y}_n]$, the total least-squares (TLS) solution for (1) is given by [31]

$$\mathbf{w}_n = -\frac{[z_{n,1}, z_{n,2}]^T}{z_{n,3}} \tag{2}$$

where $[z_{n,1}, z_{n,2}, z_{n,3}]^T$ is the right singular vector corresponding to the smallest singular value of $[\mathbf{X}_n^T, \mathbf{y}_n]$ or the eigenvector corresponding to the smallest eigenvalue of

$$\boldsymbol{\Psi}_n = \begin{bmatrix} \mathbf{X}_n \\ \mathbf{y}_n^T \end{bmatrix}^* [\mathbf{X}_n^T, \mathbf{y}_n].$$

The solution of (2) is optimal. However, obtaining it comes at the expense of updating and performing eigendecomposition of the 3×3 matrix, $\boldsymbol{\Psi}_n$, at each iteration. In the light of the analysis of [31], a computationally more efficient alternative approach can be devised by minimizing the following cost function over $\mathbf{w} \in \mathbb{C}^{2\times 1}$:

$$\mathcal{J}_n(\mathbf{w}) = \frac{\left\| [\mathbf{X}_n^T, \mathbf{y}_n] \begin{bmatrix} \mathbf{w} \\ -1 \end{bmatrix} \right\|^2}{\left\| \begin{bmatrix} \mathbf{w} \\ -1 \end{bmatrix} \right\|^2},$$