Letters

Average Current-Mode Control Scheme for a Quadratic Buck Converter With a Single Switch

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***Abstract*—A controller for a quadratic buck converter is given using average current-mode control. The converter has two *LC* filters; thus, it will exhibit fourth-order characteristic dynamics. The proposed scheme employs an inner loop that uses the current of the first inductor. This current can also be used for overload protection; therefore, the full benefits of current-mode control are maintained. For the outer loop, a conventional controller which provides good regulation characteristics is used. The design-oriented analytic results allow the designer to easily pinpoint the control circuit parameters that optimize the converter's performance. Experimental results are given for a 28 W switching regulator where current-mode control and voltage-mode control are compared.**

***Index Terms*—Current-mode control, quadratic converters, switch-mode dc–dc converters, voltage-mode control.**

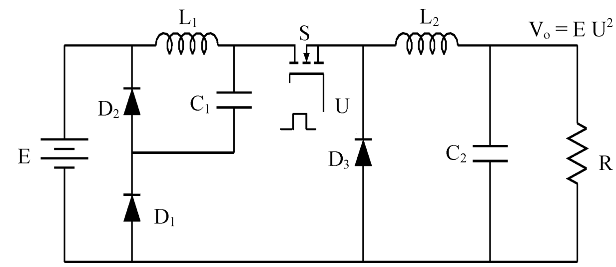

Fig. 1. Quadratic buck converter with a single switch.

I. Introduction

DURING the last two decades, a great number of applications for dc–dc converters have been reported [1], [2]. In recent years, new technologies which require wider conversion ratios have emerged; for example, new integrated circuits are using 3.3 V or 1.5 V power supplies. Also, the automotive industry is moving from 14 V (corresponding to a 12 V lead-acid storage battery) to 42 V (corresponding to a 36 V lead-acid storage battery). The above is due to the electric-electronic load in automobiles, which has been growing rapidly and is starting to exceed the practical capacity of the electric system [3].

A scheme that provides wider conversion ratios is the cascade connection of converters [4]. This scheme is a multistage approach that consists of two or more converters connected in cascade. One of the main disadvantages of the cascade connection is that the total efficiency is reduced, mainly, by losses in the switching devices. If a quadratic ratio is required, it is much better to use quadratic converters. These use only one active switch. Matsuo and Harada [5] proposed the cascade connection of buck and buck-boost converters to obtain low-voltage in power supplies. Maksimovic and Cuk [6] proposed several topologies of quadratic converters and discussed the dc operating conditions only. Pacheco *et al.* [7] proposed a buck converter with a quadratic dc conversion ratio, high switching frequency, and lossless commutation. In a quadratic buck converter with a single switch, the dc conversion ratio has a quadratic dependence on the duty ratio, and it is electrically equivalent to two-buck converters connected in cascade but using one active switch. The presence of two *LC* filters will result in a fourth-order dynamics, which will also appear when an *LC* filter is employed in a typical converter [8] between the output of the converter module and the load. Although, quadratic converters have been reported in the open literature, it is difficult to find literature that discusses the design of controllers for such a class of converters. Current-mode control has been proposed for a quadratic buck converter where the current of the first inductor was used for feedback purposes [9]. Controller design has been discussed in [10], [11] for two-stage dc–dc converters where a buck converter is used in the first stage and an isolated buck converter for the second stage.

The purpose of this letter is to give a methodology for controller design, under average current-mode control, of a quadratic buck converter with a single switch. It is relevant to establish that this control technique has been widely used for the control of dc–dc converters where the effect of the current loop in the dynamic behavior has been documented. This effect is reflected in the change from a second-order dynamics to a dominant first-order dynamics. However, this behavior is not clear in more complex topologies, as in the case of quadratic converters due to inner coupling, or in the presence of complex right-half plane (RHP) zeros.

The remainder of this letter is organized as follows. In Section II, a brief description about the modeling of the converter and the analysis of corresponding transfer functions are given. A procedure for controller design is presented in Section III, which is based on simple criteria for the loop gain. Experimental results are shown in Section IV, which confirm the results of the above analysis and design. Section V concludes this letter with some final remarks.

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II. Modeling of the Converter

The scheme of the quadratic buck converter with a single switch is given in Fig. 1. This circuit contains two LC filters, one active switch and three passive switches. The main advantage of this converter is the use of one MOSFET control circuit, in comparison with a conventional cascade converter, which requires two control circuits. In this converter, the dc voltage gain is a quadratic function of the nominal duty ratio U, i.e., $V_o = U^2 E$.

The nominal steady-state operation point of the converter can be easily derived resulting in the following expressions: $V_{\rm C1} = \rm UE$, $V_{\rm C2} = U^2\rm E$, $I_{\rm L1} = U^3\rm E/R$ and $I_{\rm L2} = U^2\rm E/R$ where the output voltage $V_{\rm o}$ corresponds to $V_{\rm C2}$. Here and what follows ($\sim$) will be used to denote perturbations from equilibrium, with capital letters for values in the equilibrium.

Averaging techniques have been proposed in [12] to obtain nonlinear and linear models for this class of converters. The corresponding linear model is given by

$$\begin{bmatrix} \dot{\tilde{i}}_{L1} \\ \dot{\tilde{i}}_{L2} \\ \dot{\tilde{v}}_{C1} \\ \dot{\tilde{v}}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & 0 \\ 0 & 0 & \frac{U}{L_2} & -\frac{1}{L_2} \\ \frac{1}{C_1} & -\frac{U}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \times \begin{bmatrix} \tilde{i}_{L1} \\ \tilde{i}_{L2} \\ \tilde{v}_{C1} \\ \tilde{v}_{C2} \end{bmatrix} + \begin{bmatrix} \frac{E}{L_1} \\ \frac{UE}{L_2} \\ \frac{-U^2E}{RC_1} \\ 0 \end{bmatrix} \tilde{u}(t) \quad (1)$$

where $\tilde{u}(t)$ is the duty ratio. The process of linearization describes the converter behavior to small perturbations around an operating point.

Now by applying the Laplace transform to the small-signal model given in (1), and after some algebraic manipulations, the corresponding transfer functions G_1 (s) (first inductor current-to-duty ratio), G_2 (s) (second inductor current-to-duty ratio) and G_3 (s) (output voltage-to-duty ratio) are obtained as follows:

$$G_1(\mathrm{s}) = \frac{\tilde{i}_{L1}(\mathrm{s})}{\tilde{u}(\mathrm{s})} = \frac{V_o}{U^2L_1} \times \frac{\mathrm{s}^3 + b_2\mathrm{s}^2 + b_1\mathrm{s} + b_o}{\mathrm{s}^4 + a_3\mathrm{s}^3 + a_2\mathrm{s}^2 + a_1\mathrm{s} + a_o} \quad (2)$$

$$G_2(\mathrm{s}) = \frac{\tilde{i}_{L2}(\mathrm{s})}{\tilde{u}(\mathrm{s})} = \frac{V_o}{UL_2} \times \frac{\mathrm{s}^3 + c_2\mathrm{s}^2 + c_1\mathrm{s} + c_o}{\mathrm{s}^4 + a^3\mathrm{s}^3 + a_2\mathrm{s}^2 + a_1\mathrm{s} + a_o} \quad (3)$$

$$G_3(\mathrm{s}) = \frac{\tilde{v}_o(\mathrm{s})}{\tilde{u}(\mathrm{s})} = \frac{V_o}{UC_2L_2} \times \frac{\mathrm{s}^2 + d_1\mathrm{s} + d_o}{\mathrm{s}^4 + a^3\mathrm{s}^3 + a_2\mathrm{s}^2 + a_1\mathrm{s} + a_o} \quad (4)$$

where

$$a_3 = \frac{1}{RC_2}; \quad a_2 = \frac{U^2}{C_1L_2} + \frac{1}{L_2C_2} + \frac{1}{L_1C_1}$$

$$a_1 = \frac{U^2}{RC_1L_2C_2} + \frac{1}{RL_1C_1C_2} \quad ; \quad a_o = \frac{1}{L_1C_1L_2C_2}$$

$$b_2 = \frac{U^2}{RC_1} + \frac{1}{RC_2}$$

$$b_1 = \frac{2\mathrm{U}^2}{C_1L_2} + \frac{1}{L_2C_2} + \frac{U^2}{\mathrm{R}^2C_1C_2}$$

$$b_o = \frac{3\mathrm{U}^2}{RC_1L_2C_2} \quad ; c_2 = \frac{1}{RC_2} - \frac{U^2}{RC_1}$$

$$c_1 = \frac{2}{L_1C_1} - \frac{U^2}{R^2C_1C_2} \quad ; \quad c_o = \frac{2}{RL_1C_1C_2}$$

$$d_1 = -\frac{U^2}{RC_1} \quad ; \quad d_o = \frac{2}{L_1C_1}. \quad (5)$$

The Routh-Hurwitz method is now used to study the stability of the above transfer functions. It is easy to show that these transfer functions are stable for all possible circuit values. The zeros of G_1(s) will be located in the left-half plane (LHP) for all possible circuit values; therefore, this transfer function is minimum phase. The zeros of the transfer function G_2(s) will be located at

$$s_1 = -\frac{1}{RC_2} \quad \text{and} \quad s_{2,3} = \frac{U^2}{2RC_1} \pm \sqrt{\frac{U^4L_1^2 - 8L_1C_1R^2}{2RL_1C_1}}$$

therefore, G_2(s) is a nonminimum phase transfer function, as there are RHP zeros. For typical circuit values the RHP zeros will be complex.

For G_3(s), the numerator has a negative coefficient in the presence of two positive coefficients; therefore, there are two zeros in the RHP; thus G_3(s) is also a nonminimum phase transfer function. The corresponding zeros of G_3(s) are located at

$$s_{z1,2} = \sigma \pm \mathrm{j}\omega \quad \sigma = \frac{U^2}{2C_1R} \quad \omega = \sqrt{\frac{2}{L_1C_1} - \sigma^2}.$$

These complex zeros are located on a circle of radius $\sqrt{2/L_1C_1}$ and the corresponding real parts depend on the load. For large loads, i.e., small values of R, the zeros move further to the right making the quadratic buck converter more difficult to control. The transfer functions between second inductor current-to-duty ratio and output voltage-to-duty ratio are nonminimum phase due to turn on-off of the active switch that causes an extra delay in the signals of the circuit, as is described by Krein in [1]. The characteristic dynamics for a typical buck converter and a quadratic buck converter are very different. The former has two complex poles and no zeros; therefore, a compensator can be easily designed. However, for the quadratic buck converter, the compensator design is much different as the output voltage-to-duty ratio transfer function has two pairs of complex poles and a pair of complex RHP zeros.

III. Procedure for Controller Design

The two general approaches for control are: a) voltage-mode, and (b) current-mode. Current-mode control of switching power converters has many advantages over voltage-mode control: (a) a faster transient response, (b) easier-to-design control loop, and (c) over-current protection within one switching cycle.

For voltage-mode control, the output voltage is needed for feedback; however, the transfer function from the output voltage-to-duty ratio has a pair of RHP zeros. Stability and good closed-loop performance are very difficult to achieve with a single-loop control since the crossover frequency of the open loop gain is severely restricted by the RHP zeros. Average current-mode control is implemented by sensing an inductor current. This current is used later for feedback purposes together with the output voltage (see Fig. 2). If the second inductor current is used, the corresponding transfer function is nonminimum phase and the compensator will be difficult to design. Thus, by feeding back the first inductor current, the corresponding transfer function is minimum phase with nice properties for current-mode control. The corresponding block