Improved robust finite-horizon Kalman filtering for uncertain networked time-varying systems

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ABSTRACT

A novel robust finite-horizon Kalman filter is presented for networked linear time-varying systems with norm-bounded parameter uncertainty whether, or not, the data packets in the network are time-stamped. Measured data loss and latency in the communication link are both described by a Bernoulli distributed random sequence. Then, a two-stage recursive structure is employed for the robust Kalman filter. The filter parameters are determined such that the covariance of the estimation error does not exceed the prescribed upper bound. New augmented state-space model is employed to derive a procedure for computation of the filter parameters. The main novelty of the paper is to use the measurement reorganization technique for the robust Kalman filter design where the observation dropout and delay are both modeled by a stochastic process. Finally, the simulation results confirm the outperformance of the proposed robust Kalman filter compared to the rival methods in the literature.

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1. Introduction

In networked control systems (NCSs) as well as sensor networks, data dropout and delay occur in information exchange through communication channels. The main origin of these phenomena is the limited capacity of the communication link, which allows at any time, only a limited number of data resources to be accommodated in the network. In addition, due to power limitation or mutual interference among some types of sensors, such as radar-based devices, simultaneous usage of them is impossible, meaning that some of the observed data is lost.

A research direction in the community of NCSs for overcoming the mentioned issues is to develop a network-access strategy (or activation management policy) for the sensors and actuators to achieve the required performance. Recently, efficient control-scheduling co-design methods were proposed to guarantee the desired performance [4,6,9]. Control synthesis was integrated with event-driven assignment schemes for numerous practical networked systems having actuators and sensors be driven by stochastic events [5,7,8]. On the other hand, state estimation from the data transmitted via unreliable communication is utilized for implementation of feedback control, monitoring and fault diagnosis of networked processes even when uncertain models of the systems exist. The Kalman filter, as a popular tool for state estimation through the noisy observations, has been extended for the uncertain networked systems with intermittent observations [12,15,23–25].

Measurement reorganization is an effective method to handle the measurement delays. It reconstructs an output delayed system in the form of its equivalent delay-free counterpart. In [24,25], the optimal estimator was designed for discrete-time

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systems with bounded time-varying delay using reorganized innovation approach. An alternative method was presented in [23] to develop a suboptimal filter in the linear minimum variance sense with a new criterion. In [2], the arrival of the inter-mittent observations was modeled as a random process investigating the stability of the Kalman filter with respect to the data arrival rate. Random delays and packet dropouts were modeled employing two groups of Bernoulli random variables in [15] to build linear estimator for networked control systems. The Kalman filter was extended in [12], for systems with random sensor delays, packet dropouts and missing measurements. Random delays and measurement losses were modeled utilizing a group of Bernoulli distributed random variables in [14]; then, an optimal linear filter depending on the probabili-ties was presented in the linear minimum variance sense. The method introduced in [14] was improved recently in [13] using a two-stage structure for the Kalman filter.

On the other hand, robust Kalman filter for systems with time-varying norm-bounded uncertainties in the state and out-put matrices were proposed in [3,16–18,21,26]. To the best of the authors' knowledge, the general problem wherein delayed and lost observations are combined with the norm-bounded uncertainties was investigated only in [10,11,19,22]. In [19], Kalman filter was designed for linear uncertain systems with randomly varying sensor delays. An upper bound was com-puted to the steady-state error covariance and filter gain was obtained by solving Riccati-like matrix inequalities. Robust Kal-man filter was derived in [10,11] for the uncertain systems including single delayed measurements based on the reorganization of the measurements. In [22], robust filtering problem was solved for discrete time-varying systems with delayed sensor measurement, in which a Riccati difference equation was developed to design the filter parameters such that an upper bound for the state estimation error variance was guaranteed.

In this paper, robust finite-horizon Kalman filter is developed for uncertain linear time-varying system where the system output is received via a packet-delaying lossy network. The phenomena of random transmission delay and multiple data dropouts are described using Bernoulli-distributed random sequence. The filter is developed for two cases: (1) when the data packets in the network are time-stamped, i.e., the filter has the knowledge of the packet delay and dropout values and (2) when they are not, i.e., the estimator does not have any information about the exact value of the packet delay and dropout but their probabilities. The main novelty of the proposed method compared to [10,11,19,22] is employing the measurement reorganization technique where the varying observation delay and loss are modeled by Bernoulli-distributed random variables. Moreover, a two-stage recursive structure is used for the robust Kalman filter. Furthermore, a new augmented state-space model is utilized to compute the filter parameters.

The rest of the paper is organized as follows: The estimation problem is formulated in Section 2. In Section 3, the optimal estimator is derived for the systems with/without time-stamped packets, respectively. Numerical comparative examples are presented in Section 4 to illustrate the merits of the proposed approach. Section 5 concludes this note.

Notations: $\Re$ denotes real numbers set. E{} is the mathematical expectation. The superscripts ^{T} and † stand for the matrix transposition and Moore–Penrose inverse of matrix [1]. diag{} represents the diagonal matrix of the elements in the braces.

2. Problem setup

Consider the following class of uncertain linear discrete-time stochastic system with the measurement equation as:

$$x(k + 1) = (A_k + \Delta A_k)x(k) + B_k w(k) \tag{1}$$
$$z(k) = C_k x(k) + v(k) \tag{2}$$

where $x(k) \in \Re^n$ is the state vector, $z(k) \in \Re^m$ is the measured output, $w(k) \in \Re^n$ and $v(k) \in \Re^m$ are the process and measure-ment noises, respectively. $w(k)$ and $v(k)$ are assumed to be uncorrelated white noises with zero means and variances Q_k and $\tilde{R}_k$. The matrices A_k, B_k and C_k are real and time-varying. ΔA_k is a real-valued uncertain matrix satisfying:

$$\Delta A_k = H_k F_k E_k, \quad F_k F_k^T \le I \tag{3}$$

H_k and E_k are known time-varying matrices of appropriate dimensions and F_k denotes time-varying uncertainties.

The sensor measurement, $z(k)$ sent to the estimator, may be delayed or lost. The sum of the largest delay and consecutive packet dropouts do not exceed N time steps where the upper bound N is known and $N(k) \le k$. The received data by the esti-mator, $y(k)$ are modeled as follows:

$$y(k) = \gamma_0(k)z(k) + (1 - \gamma_0(k))\gamma_1(k)z(k - 1) + \cdots + \prod_{i=0}^{N-1}(1 - \gamma_i(k))\gamma_N(k)z(k - N) \tag{4}$$

where $\gamma_i(k)$, $i = 0, 1, \ldots, N$ are mutually uncorrelated scalar random variables having Bernoulli distribution with mean ϖ_i, $i = 0, 1, \ldots, N$ and $0 \le \varpi_i \le 1$, i.e. prob$\{\gamma_i(k) = 1\} = \varpi_i$, prob$\{\gamma_i(k) = 0\} = 1 - \varpi_i$. It is assumed that $\gamma_i(k)$, $i = 0, 1, \ldots, N$ is uncorrelated with $w(k)$ and $v(k)$.

To clarify the data transmission model, a typical scenario is explained in Table 1 with $N = 2$, wherein $z(1)$, $z(3)$ and $z(6)$ are received on-time; $z(2)$ and $z(7)$ are lost; $z(4)$ is delayed one step and $z(5)$ is delayed two steps. Since the output packets are sent several times at each moment to establish a reliable communication, $z(1)$ is re-received at $k = 2$. The corresponding val-ues of $\gamma_i(k)$, $i = 0, 1, 2$ are summarized in Table 1 based on the Eq. (4).