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Kalman filter based fault detection for two-dimensional systems

Zhenheng Wang, Helen Shang*

School of Engineering, Laurentian University, Sudbury, Ontario, Canada P3E 2C6

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ABSTRACT

Fault detection and isolation for two-dimensional (2-D) systems represent a great challenge in both theoretical development and applications. Use of Kalman filters in fault detection has been well developed for one-dimensional (1-D) system. In this paper, a fault detection algorithm is developed for 2-D systems described by the Fornasini & Marchesini (F–M) model. Based on the state estimate from a recursive 2-D Kalman filter, a residual is generated. From the model of residual over a 2-D evaluation window, the residual is explicitly related to faults within the evaluation window. Using generalized likelihood ratio (GLR), residual evaluation is carried out and compared with a calculated threshold value for fault detection. Simulation results indicate that faults can be detected effectively using the proposed method.

1. Introduction

Fault detection and isolation (FDI) are essential in ensuring safe operations by providing timely diagnostic information to plant operators in process industry. Extensive research has been carried out on fault detection and diagnosis ranging from analytical methods to artificial intelligence and statistical approaches [1,2]. The analytical schemes for fault detection and diagnosis are basically signal processing techniques using state estimation, parameter estimation and adaptive filtering. The key component in these schemes is to design diagnostic observers with satisfactory decoupling properties for residual generation of dynamic systems. For systems where variables display noisy fluctuations with known statistical parameters, fault diagnosis problem entails monitoring the innovation process or the prediction errors [3,4]. A Kalman filter, designed on the basis of the system model in its normal operating mode, provides a state estimator with minimum estimation error and has been used for the fault detection and isolation purposes [5,6]. The well-developed fault detection and diagnosis techniques have been predominantly focused on 1-D system.

Many industrial systems are, however, two-dimensional (2-D) systems, where state variables vary in both time and space (e.g., tubular reactors and sheet forming). Several 2-D state space models were presented in the 1970s to described the 2-D systems, including the Fornasini & Marchesini [7], the Attasi [8], and the Roesser models [9]. Since then, the Fornasini & Marchesini (F–M model) and the Roesser models have been commonly adopted to describe linear discrete-time discrete-space 2-D systems. Extensive research has been carried out on 2-D systems in the past 30 years. The 2-D transfer functions, controllability and observability were defined for the F–M model and Roesser model [10,11]. The model representations such as ARX and ARMAX in one-dimensional (1-D) notation were extended to 2-D cases [12]. Identification techniques for 2-D systems were investigated using least square [13] and subspace identification [14]. A variety of observers have been proposed for 2-D state estimation [15–17]. Realization methods have been developed for realizing 2-D transfer functions to state space models [18–22]. Active interest has also been focused on filters [23–28] and control design [29–32] of 2-D systems.

Research and development on FDI of 2-D systems have been limited, owing in part to complexity of 2-D models [33–35]. The dead-beat observer has recently been applied to fault detection and isolation of 2-D systems by constructing a residual generator [36–38]. It has been shown that the dead-beat observer based FDI is effective for 2-D systems but strict conditions are required in order for an observer and a residual generator to exist. These strict conditions may limit the applicability of the existing methods.

Kalman filter based fault detection methods have been well accepted in 1-D cases. Since 1970s, extensive research has been involved with the attempt to introduce 2-D Kalman filters [39–42]. A straightforward extension of 1-D Kalman filter techniques would result in a number of state variables proportional to N for the filtering of an $N \times N$ digital image [43]. In a 2-D case, the enormous quantity of the data calls for an efficient recursion processor. In parallel with the active research on development of efficient 2-D Kalman filters, it is of great interest to explore the applications of 2-D Kalman filters to fault detection of 2-D systems.

* Corresponding author. Fax: +1 705 675 4862.
E-mail address: hshang@laurentian.ca (H. Shang).

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In this paper, a Kalman filter based fault detection method is developed for 2-D systems described by F–M models. The state estimate minimizing the estimation error variance is recursively calculated using a 2-D Kalman filter. The obtained state estimate leads to generation of a residual through the innovation process. The generated residual is of zero mean Gaussian noise when there exists no fault, and is no longer zero mean Gaussian noise when there are faults. From formulation of the residual model over a 2-D evaluation window, the residual is explicitly related to the faults in the evaluation window. The residual directly reflects the fault information and can therefore be used to determine whether a fault occurs. Residual evaluation is performed and compared with a threshold value obtained from applying the generalized likelihood ratio (GLR) method. Simulations were carried out to investigate the performance of the proposed fault detection method. Results indicate that the generated residual responds to occurrences of faults and faults can be detected by comparing the residual evaluation function with a threshold value. The proposed method is shown to be effective in fault detection of 2-D systems.

The main advantage of the proposed Kalman filter based FDI method over the dead-beat observer based FDI lies in the fact that the dead-beat observer based FDI requires strict conditions and thus has very limited applications while the Kalman filter based FDI can be applied to a much wider range of systems. In the dead-beat observer based FDI, it is required that some 2-D polynomial matrices, such as 2-D observability PBH matrix, be zero right prime in order for an observer and a residual generator to exist. Zero right primeness of the polynomial matrices may not hold for some systems and these strict conditions limit the applicability of the observer based FDI methods. For the Kalman filter based approach, however, a filter can always be constructed for a system with given initial conditions and it is, therefore, an approach with wider applicability.

2. 2-D Kalman filter based fault detection

In the Kalman filter based fault detection, a residual signal is generated using the state estimate obtained from a Kalman filter. Due to the noisy characteristics of systems, it is necessary to examine the residual over a 2-D evaluation window and perform fault detection using a statistical approach.

2.1. Residual generation

A recursive Kalman filter is used to derive a state estimate and the residual is then calculated based on the state estimation and measured output. The structure of the 2-D Kalman filter based residual generator is illustrated in Fig. 1. Note that when a fault signal is introduced, a variation in output would be observed even without using the Kalman filter. On the other hand, output variations are not necessarily caused by faults as they may be generated due to known inputs, disturbances, noises or unknown faults. It is thus impossible to determine whether a fault has occurred solely from output variations. A Kalman filter is included in the proposed fault detection method in order to calculate the residual, which is then evaluated to determine whether a fault has occurred.

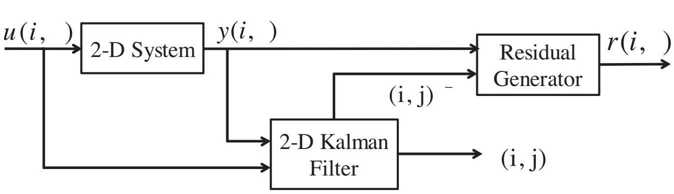

Fig. 1. Structure of a 2-D Kalman filter based residual generator.

Consider a 2-D F–M model with a fault signal and process noises:

$$\begin{aligned} x(i+1,j+1) &= A_1x(i+1,j)+A_2x(i,j+1)+B_1u(i+1,j) \\ &+B_2u(i,j+1)+F_1f(i+1,j)+F_2f(i,j+1)+H_1\eta(i+1,j) \\ &+H_2\eta(i,j+1), \\ y(i,j) &= Cx(i,j)+K_ff(i,j)+\nu(i,j), \end{aligned} \tag{1}$$

with the boundary conditions:

$$x(i,0)=x_{i0},\, x(0,j)=x_{0j},$$

where i and j indicate the horizontal (or spacial) and vertical (or time) indices, respectively, $x(i,j)\in\mathbb{R}^{n_x}$, $u(i,j)\in\mathbb{R}^{n_u}$, $f(i,j)\in\mathbb{R}^{n_f}$ and $y(i,j)\in\mathbb{R}^{n_y}$ are state, input, fault and output vectors, respectively, $\eta(i,j)\in\mathbb{R}^{n_\eta}$ indicates the process noise, $\nu(i,j)\in\mathbb{R}^{n_y}$ is measurement noise, $A_1, A_2, B_1, B_2, H_1, H_2, F_1, F_2, C, K_f$ are known matrices of appropriate dimensions. It is assumed that the variables $\eta(i,j)$ and $\nu(i,j)$ be white Gaussian noises with zero mean value and given variances:

$$\begin{aligned} &E[\eta(i,j)]=0, \\ &E[\nu(i,j)]=0, \\ &E[\eta(i,j)\eta^T(k,l)]=\begin{cases} Q(i,j), & \text{when } i=k \text{ and } j=l, \\ 0, & \text{elsewhere}, \end{cases} \\ &E[\nu(i,j)\nu^T(k,l)]=\begin{cases} R(i,j), & \text{when } i=k \text{ and } j=l, \\ 0, & \text{elsewhere}, \end{cases} \end{aligned} \tag{2}$$

where Q, and R are the covariance matrices of $\eta(i,j)$ and $\nu(i,j)$, respectively.

Let $\widehat{x}(i,j)$ be the estimate of $x(i,j)$ and $\widetilde{x}(i,j)$ be the estimation error, i.e., $\widetilde{x}(i,j)=x(i,j)-\widehat{x}(i,j)$. A Kalman filter is designed to update state estimate $\widehat{x}(i,j)$ such that the variance of estimation error $\widetilde{x}(i,j)$ be minimized. The variance of estimate error $\widetilde{x}(i,j)$ can be written as

$$P(i,j)=E[\widetilde{x}(i,j)\widetilde{x}(i,j)^T]. \tag{3}$$

With the given initial condition, the boundary values of $P(i,j)$ are obtained:

$$\begin{aligned} P(i,0) &= E[(x_{i0}-\widehat{x}(i,0))(x_{i0}-\widehat{x}(i,0))^T], \\ P(0,j) &= E[(x_{0j}-\widehat{x}(0,j))(x_{i0}-\widehat{x}(0,j))^T]. \end{aligned} \tag{4}$$

The proposed fault detection is to generate a residual signal such that it reflects occurrence of faults. For the residual generation, the state estimate $\widehat{x}(i,j)$ is to be updated recursively. We adopted the Kalman filter proposed in [44] due to its recursive nature and other Kalman filters should be equally applicable. The recursive scheme for optimal state estimate can be expressed:

$$\widehat{x}(i,0)=\widehat{x}_{i0}, \quad \widehat{x}(0,j)=\widehat{x}_{0j}, \tag{5}$$

$$\widehat{x}(i,j)|^- = A_1\widehat{x}(i,j-1)+A_2\widehat{x}(i-1,j)+B_1u(i,j-1)+B_2u(i-1,j), \tag{6}$$

$$\widehat{x}(i,j)=\widehat{x}(i,j)|^- + L(i,j)[y(i,j)-C\widehat{x}(i,j)|^-], \tag{7}$$

where $\widehat{x}(i,j)|^-$ indicates the state estimate at (i,j) based on measurement up to $(i-1,j)$ and $(i,j-1)$, while $\widehat{x}(i,j)$ is the state estimate at (i,j) based on measurement up to (i,j). $L(i,j)$ denotes the Kalman filter gain. Corresponding to $\widehat{x}(i,j)|^-$ and $\widehat{x}(i,j)$, the associated estimation error variance matrices are:

$$P(i,j)|^- = E[(x(i,j)-\widehat{x}|^-(i,j))(x(i,j)-\widehat{x}|^-(i,j))^T], \tag{8}$$

$$P(i,j)=E[(x(i,j)-\widehat{x}(i,j))(x(i,j)-\widehat{x}(i,j))^T]. \tag{9}$$