Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-22 (2015) 052–057

Localized Distributed Kalman Filters for Large-Scale Systems ⋆

Yuh-Shyang Wang * **Seungil You** * **Nikolai Matni** *

* *Department of Control and Dynamical Systems, California Institute of Technology, Pasadena, CA 91125, USA (e-mail: {yswang,syou,nmatni}@caltech.edu)*

Abstract: This paper presents a scalable method to design large-scale Kalman-like filters for a class of linear systems. In particular, we consider systems for which both the propagation of dynamics through the plant and the exchange of information between estimators/sensors is subject to delays. Under suitable assumptions on these delays, our proposed Kalman-like filter has the following desirable properties: (1) each local estimator only needs to collect the information within a localized region to estimate its local state, and (2) each local estimator can be designed by solving a local optimization problem using local plant model information. The decomposition of the global problem into local subproblems thus allows for the method to scale to arbitrarily large heterogeneous systems – this is clearly an extremely favorable property for large-scale estimation problems. The effectiveness of our algorithm is demonstrated on a randomized heterogeneous example with 51200 states, in which the traditional Kalman filter cannot be computed within a reasonable amount of time.

© 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Kalman filters, State estimation, Optimal estimation, Large-scale systems, Distributed control

1. INTRODUCTION

The celebrated Kalman filter achieves minimum mean square error for linear state estimation problems via an elegant and easily interpretable recursive method. Unfortunately, the Kalman filter is an inherently centralized method, and is neither scalable to compute nor physically implementable for large-scale systems. Specifically, the computation of the traditional Kalman filter involves solving an Algebraic Riccati Equation (ARE) and computing a matrix inverse, which is complicated when the size of the problem goes large. Even if a centralized estimator can be computed, large-scale estimation problems are nonetheless subject to practical communication delays between sensors and estimators which can degrade the performance of a centralized scheme substantially. These limitations make centralized estimation unappealing in large-scale applications such as weather forecasting (Farrell and Ioannou (2001)), ocean data assimilation (Fukumori and Malanotte-Rizzoli (1995)), biological signal analysis (Long et al. (2006)), and state estimation in the power grid (Huang et al. (2012)).

Various methods have been proposed in the field of distributed Kalman filtering, but many still suffer from scalability issues that limit their application to large-scale systems. For instance, both the consensus-based algorithm of Olfati-Saber (2007) and the diffusion-based algorithm of Cattivelli and Sayed (2010) require each local sensor to store and use the *global plant model*, and to estimate the *global state* during implementation. This introduces huge computational burden, and is prohibitive for large-scale applications. An exception is the work of Khan and Moura (2008), in which the authors use spatial decomposition, observation fusion, and approximated algorithms on matrix inversion to design a scalable Kalman-like filter. However, the algorithms involve multiple iterations, and the transient behavior of the algorithm is hard to analyze.

In this paper, we propose a scalable method to design large-scale Kalman-like filters based on the notion of *localizability for state estimation*. This notion can be viewed as a generalization of observability, in which the state estimator is restricted to a subspace dictated by spatiotemporal constraints. Intuitively, when information can be shared sufficiently quickly among local estimators, the uncertainty in the state due to local process and sensor noise can be isolated to a localized region. In other words, the closed loop response from process and sensor noise to the estimation error is *localized*. We show that finding such a localized closed loop response, if it exists, can be done in an efficient and scalable manner, and further demonstrate that the resulting Kalman-like estimator can be *designed* and *implemented* in a localized way. Our main technical tool is the duality that exists between Kalman Filtering and Linear Quadratic Regulation (LQR), and use this duality to formulate the localized distributed Kalman filter (LDKF) problem as a localized LQR (LLQR) problem (Wang et al. (2014) and Wang and Matni (2015)).

⋆ This research was in part supported by NSF NetSE, AFOSR, the Institute for Collaborative Biotechnologies through grant W911NF-09-0001 from the U.S. Army Research Office, and from MURIs "Scalable, Data-Driven, and Provably-Correct Analysis of Networks" (ONR) and "Tools for the Analysis and Design of Complex Multi-Scale Networks" (ARO). The content does not necessarily reflect the position or the policy of the Government, and no official endorsement should be inferred.

2405-8963 © 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.10.306

The rest of this paper is structured as follows. In Section 2, we review the traditional Kalman filter and emphasize its limitations in the context of large-scale systems. In Section 3, we reformulate the Kalman filter problem as an optimization problem over the *closed loop transfer matrices.* We then introduce the idea of localizability for state estimation by imposing an additional spatiotemporal constraint on the closed loop transfer matrices, and formulate the LDKF problem in Section 4. Specifically, LDKF estimator can be designed and implemented in a localized and parallel way using techniques developed to solve the LLQR problem, as described in Wang et al. (2014). To demonstrate the effectiveness of our method, we synthesize the LDKF estimator for a system with 51200 states in Section 5. Finally, conclusions are given in Section 6.

2. TRADITIONAL KALMAN FILTER

This section introduces the system model and the traditional Kalman filter. We then explain the limitations of traditional Kalman filter on large-scale systems, which motivate our work.

2.1 System Model

Consider a discrete time linear time invariant (LTI) system with dynamics given by

$$\begin{aligned} x[k+1] &= Ax[k] + Bu[k] + w[k] \\ y[k] &= Cx[k] + v[k] \end{aligned} \tag{1}$$

where x is the state, u the control input, y the sensor measurements, w the process noise, and v the sensor noise. Our goal is to design a state estimator $\hat{x}$ based on the measurement y and a pre-specified control input u. In particular, we are interested in the case when the system matrices (A, B, C) are high-dimensional yet suitably sparse. Our approach is to exploit the sparsity of (A, B, C) to derive a scalable algorithm for state estimator design.

We adopt to the common setting for Kalman filter (cf. Anderson and Moore (2012)). Let $\mathbb{E}(\cdot)$ be the expectation operator and δ_{ij} the Kronecker delta function. We assume that the process noise w and sensor noise v are independent zero mean additive white Gaussian noise (AWGN), with covariance matrices given by $\mathbb{E}(w[i]w[j]^\top) = \delta_{ij}W$, $\mathbb{E}(v[i]v[j]^\top) = \delta_{ij}V$, and $\mathbb{E}(w[i]v[j]^\top) = 0$. We assume that w and v are uncorrelated to keep the formulas simple, while the method described in this paper still works when w and v are correlated. The initial condition $x[0]$ is also assumed to be a Gaussian random vector with mean x_0 and variance Σ_0, and $x[0]$ is uncorrelated with $w[k]$ and $v[k]$ for all k.

2.2 Traditional Kalman Filter

Let $\hat{x}[k|s]$ denote the estimate of the state $x[k]$ given the collected information up to time s, i.e. the measurements $y[t]$ and control inputs $u[t]$ from $t = 1, \dots, s$. The Kalman filter for the LTI system (1) is specified by

$$\hat{x}[k|k] = \hat{x}[k|k-1] + K(y[k] - C\hat{x}[k|k-1]) \tag{2}$$

$$\hat{x}[k+1|k] = A\hat{x}[k|k] + Bu[k] \tag{3}$$

with initial condition given by $\hat{x}[0|-1] = x_0$. The matrix K in (2) is known as the Kalman gain, which can be found by solving an ARE. Let Σ be the solution to the discrete time ARE

$$\Sigma = A\Sigma A^\top + W - A\Sigma C^\top (C\Sigma C^\top + V)^{-1} C\Sigma A^\top. \tag{4}$$

The Kalman gain in (2) can then be computed as

$$K = \Sigma C^\top (C\Sigma C^\top + V)^{-1}. \tag{5}$$

The Kalman filter is optimal in the sense of minimum mean square error. Let $\tilde{x}[k|k-1] = x[k] - \hat{x}[k|k-1]$ be the estimation error before $y[k]$ is measured. The Kalman filter algorithm in (2) - (3) minimizes the mean square error

$$\mathbb{E}\left(\frac{1}{N}\sum_{k=1}^{N} \tilde{x}[k|k-1]^\top \tilde{x}[k|k-1]\right) \tag{6}$$

for $N \to \infty$. Similarly, let $\tilde{x}[k|k] = x[k] - \hat{x}[k|k]$ be the estimation error after $y[k]$ is measured. The mean square error of $\tilde{x}[k|k]$ is also minimized.

Equations (2) and (3) can be combined into a single equation as

$$\hat{x}[k+1|k] = A\hat{x}[k|k-1] + Bu[k] + L(y[k] - C\hat{x}[k|k-1]) \tag{7}$$

with $L = AK$ is a gain matrix. We refer to (7) as the *delayed form* of state estimation. We can also combine equations (2) and (3) to obtain

$$\hat{x}[k+1|k+1] = (I - KC)(A\hat{x}[k|k] + Bu[k]) + Ky[k+1]. \tag{8}$$

We refer to (8) as the *current form* of state estimation.

2.3 Limitations

Here we point out some limitations of the traditional Kalman filter for large-scale systems.

(1) The Kalman gain given by (5) is generally dense even when the system matrices (A, B, C) that specify the system dynamics (1) are sparse. This means that the measurements from all sensors need to be shared *instantaneously*, which requires infinite (or impractically fast) communication speed.
(2) A dense Kalman gain (5) also implies that the measurements from all sensor need to be collected by *every estimator in the network*, which is not scalable to implement.
(3) To compute the Kalman gain (5), one need to solve a large-scale ARE (4). The complexity of solving (4) is $O(n^3)$, where n is the dimension of the matrix A. This can be prohibitive for a large n.
(4) When the global plant model (A, B, C) changes locally, one needs to recompute the solution to (4) to resynthesize the global Kalman filter. This is not scalable for incremental design when the physical system expands.

To design a state estimation algorithm for large-scale systems, one must overcome the aforementioned limitations of the traditional Kalman filter. Distributed Kalman filter architectures in Olfati-Saber (2007) or Cattivelli and Sayed (2010) may resolve the first two limitations, but not the latter two. This motivates our development of the LDKF architecture, in which the estimator can be both implemented *and* designed in a localized and scalable way.

دریافت فوری ← متن کامل مقاله

ISIArticles
مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات