Monte Carlo Simulation for Reliability Analysis of Emergency and Standby Power Systems

Chanan Singh, Fellow, IEEE   Joydeep Mitra, Student Member, IEEE

Department of Electrical Engineering
Texas A & M University
College Station, Texas 77843

Abstract

This paper describes a sequential Monte Carlo simulation method for the reliability analysis of standby and emergency power systems. The results obtained from this method are compared with those from the Markov cut-set approach. It is shown that the Monte Carlo simulation can yield additional useful information on the probability distribution of indices in addition to obtaining the estimates of the mean values.

1 Introduction

Different facilities have varying requirements for reliability of electric power supply. Even at the same facility various loads may have different reliability requirements. Some loads, such as medical facilities, emergency lighting, data processing and chemical process industries are very sensitive to interruptions in electric supply. Standby and emergency power systems [1, 2, 3] are installed at such premises to provide electric power of acceptable quality.

Reliability and cost considerations play an important role in the choice of various alternatives. These alternatives include less expensive utility supply enhanced by standby power, more expensive utility supply, and various configurations of standby systems. The analysis of these alternatives may become more important as the reliability differentiated power becomes available.

Reliability analysis of various options is important for the proper selection of standby power systems. Although this is an important problem, it has not been adequately addressed in the available literature. References [4] and [5] describe methods based on *Markov* and *Markov cut set* approaches. This paper describes a *Monte Carlo* [6] approach for this problem.

2 Background

Reference [5] outlines an approach based on a combination of the cut-set method and the Markov models. This approach consists in first identifying cut-sets which are basically components or events whose failure or occurrence would cause system failure. The equations for calculating the failure frequency and duration of these cut-sets are described in [3, 5]. Some of the events which involve dependent failures are analyzed using Markov processes [5, 6, 7]. The methodology for combining the frequency and duration of cut-sets to obtain system indices is described in [6, 7]. This approach is quite powerful, but may run into problems of dimensionality where large and complex configurations are involved.

3 Monte Carlo Approach

The reliability indices of an actual physical system can be estimated by collecting data on the occurrence of failures and the durations of repair. The Monte Carlo method mimics the failure and repair history of the components and the system by using the probability distributions of the component state durations. Statistics are then collected and indices estimated using statistical inference.

There are two basic approaches for Monte Carlo simulation, (1) sequential simulation, and (2) random sampling. The sequential simulation proceeds by generating a sequence of events using random numbers and probability distributions of random variables representing component state durations. In random sampling, states are drawn based on the probability distributions of component states and random numbers. Further, there are two methods for representing the passage of time in sequential simulation: (1) the fixed interval method, also called synchronous timing, and (2) the next event or asynchronous timing method. In the fixed interval method, time is advanced in steps of fixed length and the system state is updated. In the next event method, time is advanced to the occurrence of the next event. In actual implementations, it is likely that combinations of the timing controls may be used.

The sampling method is generally faster than the sequential technique, but is suitable when component failures and repairs are independent. This paper presents the sequential method for reliability analysis.

3.1 Description of the Method

The flowchart for this method is shown in FIGURE 1. The whole procedure consists of the following steps.

3.1.1 Data Input and Initialization

The input data consists of the *failure rate* (λ) and *duration* (r) of every component. The failure rate is the reciprocal of the *mean up time*. The *failure duration* or *mean down time* is the reciprocal of the *repair rate* (μ). The failure and repair rates, λ and μ, of a component will be used to determine how long the component will remain in the "UP" state and the "DOWN" state.

Simulation could be started from any system state, but it is customary to begin simulation with all the components in the "UP" state.

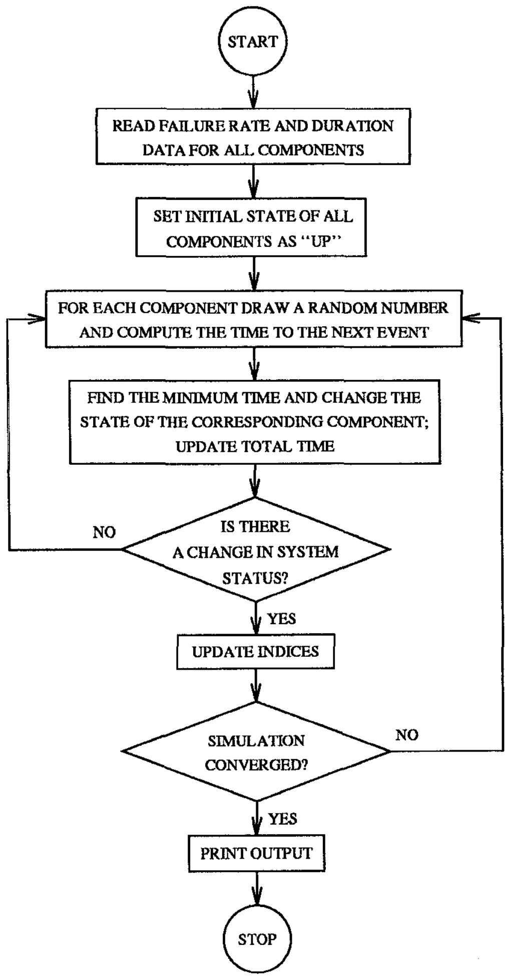

FIGURE 1: FLOWCHART FOR NEXT-EVENT SIMULATION

3.1.2 Random Number Generation

Simulation is performed by generating realizations of the underlying stochastic process, by using random numbers. These numbers constitute a sequence in which each number has an equal probability of assuming any one of the possible values, and is statistically independent of the other numbers in the sequence. Random numbers, therefore, basically constitute a uniform distribution over a suitably selected range of values. This distribution may be constructed using any suitable means. The method used in this work is a multiplicative congruential method [6] which obtains the $(n+1)$th random number R_{n+1} from the nth random number R_n using the following recurrence relation due to Lehmer

$$R_{n+1} = (aR_n)(\text{modulo } m) \tag{1}$$

where a and m are positive integers, $a < m$. The above notation signifies that R_{n+1} is the remainder when aR_n is divided by m. The first random number R_0 (called the seed) is assumed, and the subsequent numbers can be generated by the above recurrence relation. Now the sequence thus generated is periodic, so R_0, a and m should be carefully chosen so that the sequence cycle is larger than the number of random numbers required.

3.1.3 Computation of Time to the Next Event

The time to the next event is generated by using the *inverse of probability distribution* method. This method can be understood by considering the probability mass function of a random variable shown in FIGURE 2. The first step is to convert this mass function into the corresponding distribution function, as shown. Now a random number z between 0 and 1 is generated and $F(x)$ is set equal to z. The corresponding value of X gives the value of the random variable. An example is shown in FIGURE 2, with $z = 0.55$, for which $X = 2$.

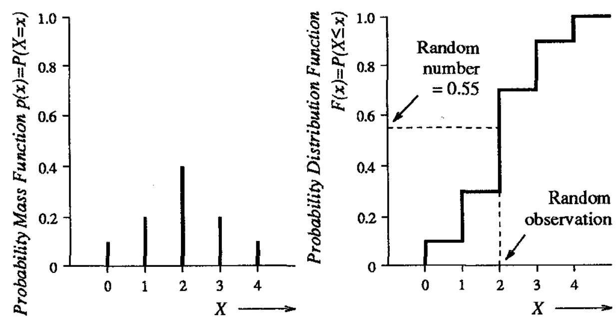

FIGURE 2: PMF AND PDF OF A RANDOM VARIABLE

It should be noticed that $F(x_i) - F(x_{i-1})$ is equal to $P(X = x_i)$, and if the random number falls in the interval $(F(x_i), F(x_{i-1}))$, the value of $X = x_i$ will be selected. The procedure therefore essentially allocates the random numbers to the random variables in the proportion of their probabilities of occurrence.

This procedure can also be used for continuous distributions. Continuous distributions are approximated by discrete distributions whose irregularly spaced points have equal probabilities. The accuracy can be increased by increasing the number of intervals into which (0,1) is divided. This requires additional data in the form of tables. Although the method is quite general, its disadvantages are the great amount of work required to develop tables and possible computer storage problems. The following analytic inversion approach is simpler.

Let z be a random number in the range 0 to 1 with a uniform probability density function, i.e., a triangular distribution function:

$$f(z) = \begin{cases} 0 & Z < 0 \\ 1 & 0 \le Z \le 1 \\ 0 & Z > 1 \end{cases} \tag{2}$$