A Phase-Domain Synchronous Machine Model With Constant Equivalent Conductance Matrix for EMTP-Type Solution

Liwei Wang, *Member, IEEE*, and Juri Jatskevich, *Senior Member, IEEE*

(Invited Paper)

***Abstract*—Interfacing machine models in either nodal analysis-based (EMTP-like) or state variable-based transient simulation programs play an important role in numerical accuracy and computational performance of the overall simulation. As an advantageous alternative to the traditional qd models, a number of advanced phase-domain (PD) and voltage-behind-reactance machine models have been recently introduced. However, the rotor-position-dependent conductance matrix in the machine–network interface complicates the use of such models in EMTP. This paper focuses on achieving constant and efficient interfacing circuit for the PD synchronous machine model. It is shown that the machine conductance matrix can be formulated into a constant submatrix plus a time-variant submatrix. Eliminating numerical saliency from the second term results in a constant conductance matrix of the proposed PD model, which is a very desirable property for the EMTP solution since the refactorization of the network conductance matrix at every time step is avoided. Case studies demonstrate that the proposed PD model represents a significant improvement over other established models used in EMTP while preserving the accuracy of the original/classical PD model.**

***Index Terms*—Constant conductance matrix, EMTP, G matrix, phase-domain (PD) model, *qd* model, saliency elimination, synchronous machine, voltage-behind-reactance (VBR) model.**

NOMENCLATURE

Throughout this paper, bold font is used to denote matrix and vector quantities, and italic nonbold font is used to denote scalar quantities.

$\mathbf{v}_{abcs}, \mathbf{v}_{qdr}$	Stator and rotor voltage vectors.
$\mathbf{i}_{abcs}, \mathbf{i}_{qdr}$	Stator and rotor current vectors.
$\boldsymbol{\lambda}_{abcs}, \boldsymbol{\lambda}_{qdr}$	Stator and rotor flux linkage vectors.
$\mathbf{R}_s, \mathbf{R}_r$	Stator and rotor diag. resistance matrices.
$\mathbf{L}_s(\theta_r), \mathbf{L}_r$	Stator and rotor self-inductance matrices.
$\mathbf{L}_{sr}(\theta_r), \mathbf{L}_{rs}(\theta_r)$	Stator and rotor mutual-inductance matrices.
θ_r	Rotor position angle.
Δt	Discretization time step.
$\mathbf{R}_{\mathrm{eq}}^{pd}, \mathbf{G}_{\mathrm{eq}}$	Equivalent resistance and conductance matrices.
$\mathbf{e}_h^{pd}, \mathbf{i}_h^{pd}$	Equivalent history voltage and current sources.
$\mathbf{e}_{sh}^{pd}, \mathbf{e}_{rh}^{pd}$	Stator and rotor equivalent history sources.
Z_q'', Z_d''	Equivalent q- and d-axis subtransient impedances.
Z_{mq}, Z_{md}	Equivalent q and d-axis magnetizing impedances.
$Z_{lj}, j = kq1, \ldots kqM, fd, kd1, \ldots kdN$	Equivalent field and damper winding impedances.
Z_q'''	PD model modified q-axis subtransient impedance with added damper winding.
R_{kqM+1}, L_{lkqM+1}	Added damper winding resistance and leakage inductance.
Z_{lkqM+1}	Added damper winding impedance.
τ_{QM+1}	Added damper winding pole time constant.
f_{fit}	Fitting frequency.
L_{critical}	Critical leakage inductance for the added damper winding.

Manuscript received June 15, 2012; revised October 18, 2012; accepted November 9, 2012. Date of publication December 3, 2012; date of current version February 7, 2013. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under the Discovery Grant, and in part by the Educational Grant from BC Hydro. Paper no. TEC-00307-2012.

L. Wang is with ABB Corporate Research Center, Västerås SE-721 78, Sweden (e-mail: liwei.wang@se.abb.com).

J. Jatskevich is with the Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, BC V6 T 1Z4, Canada (e-mail: jurij@ece.ubc.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TEC.2012.2227748

I. INTRODUCTION

MACHINE models for studying the power systems transients are generally based on the *qd*-axes model formulation. Such models have received extensive attention in the literature and are widely available in many nodal analysis-based electromagnetic transient programs (i.e., EMTP-type [1]) and state variable-based (e.g., Simulink [2]) simulation packages as built-in library components that are extensively used by many practicing engineers and researchers in industry and academia. Since such general purpose machine models are widely used, improving their numerical efficiency and accuracy can have significant impact and improve many simulation packages.

To improve the interface between the machine model and the external power system network, which is typically represented in physical abc phase coordinates, the so-called phase-domain (PD) [3]–[6] and voltage-behind-reactance (VBR) [7]–[9] models have been introduced. For numerically efficient interfacing of induction and synchronous machine models in state-variable transient simulation programs, a unified constant parameter *RL*-branch equivalent circuit has been recently proposed in [10].

Improving the numerical efficiency and accuracy of the synchronous machine models for the EMTP-type programs has been attractive for a long time. Numerous machine models have been proposed and implemented in various software packages including MicroTran [11], ATP/EMTP [12], PSCAD/EMTDC [13], and EMTP-RV [14], where the qd models are typically used. The main advantage of these methods is that they result in a constant machine conductance submatrix [18]. However, predicting relatively fast electrical variables introduces interfacing errors that significantly reduce the numerical accuracy of such models, and may potentially cause convergence problem [5], [8], [9]. In EMTP implementation, the advantages of PD and VBR synchronous machine models generally come at the price of having a rotor-position/speed-dependent machine conductance submatrix, which generally requires refactorization of the entire network conductance $\mathbf{G}$ matrix at every time step as the rotor position changes. A PD and VBR induction machine models that achieve constant conductance submatrix have been recently proposed by the authors in [15]. Achieving the same property for the VBR synchronous machine is very challenging due to the saliency and structure of equations [8]. Instead, the main focus and goal of this paper is to propose a new PD synchronous machine model that also achieves a constant conductance submatrix in EMTP solution, which, to the best of the authors' knowledge, has not been proposed before. Interested reader will find particularly useful the related work summarized in [8], [10], and [15]. The properties of the proposed PD model and the additional contributions of this paper are summarized as follows.

1) This paper shows that an equivalent resistance submatrix of the discretized PD synchronous machine model can be expressed as a constant plus a rotor-position-dependent term.
2) This property is used for deriving a model that has constant machine conductance submatrix as in [15] (assuming a magnetically linear machine) using approximation of saliency technique [16].
3) The proposed constant conductance (CC-PD) model can be achieved for either salient-pole or round rotor machines with appropriately fitted parameters for required accuracy and discretization time step.
4) The proposed CC-PD model is shown to maintain very good accuracy even at large time step similar to the classical PD model.

II. Coupled-Circuit PD Machine Model

For the purpose of power systems transients, a three-phase electrical machine can be modeled by lumped-parameter coupled circuits in physical variables and abc coordinates, known in EMTP community as the PD model. Without loss of generality, this paper considers a three-phase synchronous machine with one field winding fd, and M and N damper windings in the q- and d-axes, respectively. The parameters of the example machine considered in the case studies are summarized in Appendix A. All rotor windings are assumed to be referred to the stator side by appropriate turn ratios. Motor convention is used so that the stator currents flowing into the machine have a positive sign in the voltage equations. The flux linkage of each winding is assumed to have the same sign as the current flowing in that winding.

A. Continuous-Time PD Model

To set the stage for the further derivations, the PD model in continuous time is briefly included here. The voltage equation in physical variables and phase coordinates is [17]

$$\begin{bmatrix} \mathbf{v}_{abcs} \\ \mathbf{v}_{qdr} \end{bmatrix} = \mathbf{R} \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}_{qdr} \end{bmatrix} + p \begin{bmatrix} \boldsymbol{\lambda}_{abcs} \\ \boldsymbol{\lambda}_{qdr} \end{bmatrix} \tag{1}$$

where the resistance matrix is given as

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_s & \\ & \mathbf{R}_r \end{bmatrix}. \tag{2}$$

The flux linkage equation is expressed as

$$\begin{bmatrix} \boldsymbol{\lambda}_{abcs} \\ \boldsymbol{\lambda}_{qdr} \end{bmatrix} = \mathbf{L}(\theta_r) \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}_{qdr} \end{bmatrix} \tag{3}$$

where the stator and rotor self- inductance and mutual inductance matrix may be expressed as

$$\mathbf{L}(\theta_r) = \begin{bmatrix} \mathbf{L}_s(\theta_r) & \mathbf{L}_{sr}(\theta_r) \\ \mathbf{L}_{rs}(\theta_r) & \mathbf{L}_r \end{bmatrix}. \tag{4}$$

The detailed expressions of the resistance and inductance submatrices are given in Appendix B. The induced electromagnetic torque is calculated in terms of physical variables (currents) as follows:

$$T_e = \frac{P}{2} \left[\frac{1}{2} \mathbf{i}_{abcs}^T \frac{\partial}{\partial \theta_r} \left(\mathbf{L}_s - L_{ls}\mathbf{I} \right) \mathbf{i}_{abcs} + \mathbf{i}_{abcs}^T \frac{\partial \mathbf{L}_{sr}}{\partial \theta_r} \mathbf{i}_{qdr} \right] \tag{5}$$

which can be written in expanded form given in Appendix B, (B9) for simplified implementation instead of the involved matrix product (5). The mechanical equations can be found in [8] and [17], and are not included here due to space limitation.

B. Discretized PD Model for EMTP Solution

In order to obtain numerical solution within the EMTP, the PD machine model is discretized using implicit trapezoidal rule [1]. To interface the machine, the discretized model is first expressed in the following general form [8]:

$$\mathbf{v}_{abcs}(t) = \mathbf{R}_{\text{eq}}^{pd} \mathbf{i}_{abcs}(t) + \mathbf{e}_h^{pd}(t) \tag{6}$$