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Modelling Market Power Cost in the Assessment of Transmission Investment Policies

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***Abstract*—This paper develops a mathematical tool for modelling market power cost in transmission expansion planning decisions. The mathematical modelling is based on the game theory in applied mathematics and the concept of social welfare in microeconomics. We assume the generating companies as Cournot players and the Transmission System Operators as a regulated social transmission planner. To tackle the multiple Nash equilibria problem, the concept of worst-Nash equilibrium is defined and mathematically formulated. The developed mathematical structure is a mixed-integer linear programming problem. This closed form mathematical structure can be solved efficiently using the available computational packages.**

***Index Terms*—Transmission Augmentation, Market Power, worst-Nash equilibrium**

I. Introduction

TRANSMISSION planning is complex, involving consideration of the impact of a transmission augmentation under a large number of future demand and supply scenarios. The transmission planning problem is well understood in the context of a vertically-integrated electricity industry,[1]. In principle, if the liberalised electricity market is sufficiently competitive, the same tools and techniques that have been developed for transmission planning in the context of an integrated electricity industry can be applied. However, two new issues arise: (a) The first issue is the problem of generator market power, (b) The second is coordination between generation and transmission investment. The focus of this paper is on the first issue.

Modelling market power cost in the process of transmission expansion planning is an issue which is not researched well. It is shown that additional transmission capacity can reduce the market power cost and improve competition between rival generating companies, [2]. Modelling of the market power cost in the process of transmission expansion planning is complex and it is not researched well. In [3], [4], [5] no literature is referred about the modelling of market power for economic transmission augmentation.

Reference [6] suggests two heuristic procedures for transmission augmentation. The authors use unconstrained oligopoly equilibrium for the set of producers' bids while bids from the demand side are assumed to be fixed and derived from analysis of the existing market data.

References [7] and [8] show that generators benefit from a reduction in transmission capacity. Using a simplified version of the power network in California, [9] has quantified the impact of local market power and transmission capacity.

The TEAM methodology introduced by the California ISO [10] is a good model for analysing economic-efficiency-based transmission augmentation. However, the California ISO method is an ad-hoc and heuristic method.

To the authors' best knowledge, all approaches proposed for modelling market power cost in the process of are either ad hoc or lack an efficient solution technique.

This paper derives a mixed-integer linear programming problem for assessment of transmission investment policies taking into account the market power cost. The derived mathematical structure is closed-form and it can be solved efficiently. In section II, the strategic generating companies, the worst-Nash equilibrium, and the transmission system operator are mathematically modelled. Section III includes a case study and section IV concludes this paper.

II. Mathematical Formulation

A. Strategic Generating Companies

A strategic generating company is modelled using the static leader-follower game. In this game, the leader is the generating company and the follower is the electricity market operator. The electricity market operator runs a bid-based security-constrained economic dispatch. The generating company calculates its profit using the endogenous prices and the generation quantities available from the dispatch process.

This bilevel programming problem is nonlinear and nonconvex and consequently it is very hard to be solved efficiently. This paper uses the Karush-Kuhn-Tucker optimality conditions and the disjunctive nature of complementary slackness conditions, and it converts this bilevel programming problem into a mixed-integer linear programming problem. The final mixed-integer linear programming problem is given in (1-1) and (1-2).

The bid-based security-constrained economic dispatch is formulated in (1-1). The objective of this optimisation problem is the total operating cost. The first constraint is the energy balance equation. Second and third constraints represent the generic transmission constraints. The generation

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capacity of the generating units is modelled in the fourth and fifth constraints. Finally, the last constraint models the system reference node.

$$Max\ -\sum_g\sum_n G_{g,n}MC_gGQTY_g$$
$$s.t.$$
$$\sum_n B_{m,n}t_n=\left(\sum_g G_{g,m}GQTY_g-\sum_d D_{d,m}DQTY_d\right)\forall m\longleftrightarrow u$$
$$-\sum_n H_{l,n}t_n\geq -TCAP_l\forall l\longleftrightarrow v^{t\max}$$
$$\sum_n H_{l,n}t_n\geq -TCAP_l\forall l\longleftrightarrow v^{t\min}$$
$$-GQTY_g\geq -GCAP_g\forall g\longleftrightarrow v^{g\max}$$
$$GQTY_g\geq 0\forall g\longleftrightarrow v^{g\min}$$
$$t_N=0\longleftrightarrow w$$

(1)

Where in (1), $G_{g,n}$ is the matrix of connection points of generating units, MC_g is the short-run marginal cost of generating unit g, $GQTY_g$ is the economically dispatched generation quantity, $B_{m,n}$ is the suceptance matrix, t_n is the node angle , $D_{d,m}$ is the matrix of connection points of demand units, $DQTY_d$ is the demand level, $H_{l,n}$ is the matrix transmission lines topologies, $TCAP$ is the transmission lines capacities, $GCAP_g$ is the capacity of generating units, and $u, v^{t\max}, v^{t\min}, v^{g\max}, v^{g\min}, w$ are the Lagrange multipliers of their associated constraints. A profit-maximising generating company can be formulated as in (2).

$$Max\pi_p=\sum_{g\in P}(u_g-MC_g)GQTY_g$$
$$s.t.$$
$$0\leq GCAP_g\leq GMAX_g$$
$$Max-\sum_g\sum_n G_{g,n}MC_gGQTY_g$$
$$s.t.$$
$$\sum_n B_{m,n}t_n=\left(\sum_g G_{g,m}GQTY_g-\sum_d D_{d,m}DQTY_d\right)\forall m \quad (2)$$
$$-\sum_n H_{l,n}t_n\geq -TCAP_l\forall l$$
$$\sum_n H_{l,n}t_n\geq -TCAP_l\forall l$$
$$-GQTY_g\geq -GCAP_g\forall g$$
$$-GQTY_g\geq 0\forall g$$
$$t_N=0$$

The Karush-Kuhn-Tucker optimality conditions for optimisation problem (1) (in addition to the feasibility conditions) are as follows;

Dual Feasibility:

$$-\sum_n G_{g,n}MC_g+\sum_n u_nG_{g,n}-v_g^{g\max}+v_g^{g\min}=0\forall g$$
$$-\sum_m u_mB_{m,n}-\sum_l v_l^{t\max}H_{l,n}+\sum_l v_l^{t\min}H_{l,n}=0\forall n \quad (3)$$
$$v_l^{t\max},v_l^{t\min}\geq 0\forall l$$
$$v_g^{g\max},v_g^{g\min}\geq 0\forall g$$

Complementary Slackness Conditions:

$$v_l^{t\max}\left(TCAP_l-\sum_n H_{l,n}t_n\right)=0\forall l$$
$$v_l^{t\min}\left(TCAP_l+\sum_n H_{l,n}t_n\right)=0\forall l \quad (4)$$
$$v_g^{g\max}\left(GMAX_g-GQTY_g\right)=0\forall g$$
$$v_g^{g\min}GQTY_g=0\forall g$$

The dual feasibility constraints are all linear. The complementary slackness conditions can be linearised by introducing a binary variable. The expression XY = 0 can be written as Xb + Y(1-b) = 0 where b is a binary variable. The last expression can be written as two linear equations: $-M(1-b)\leq X\leq M(1-b)$ and $-Mb\leq Y\leq Mb$. This linearisation method is used to linearise all complementary slackness conditions.

By substituting the inner optimisation problem in (2) by its equivalent linearised KKT conditions, all constraints in optimisation problem (2) are now linear. The only non-linear term is the objective function π_p.

The strong duality condition holds for the inner optimization problem.

$$\sum_g v_g^{g\max}GMAX_g-\sum_n\left(\sum_d D_{d,n}DQTY_d\right)u_n+\sum_l v_l^{t\max}TCAP_l+\sum_l v_l^{t\min}TCAP_l=-\sum_g\sum_n G_{g,n}MC_gGQTY_g\Rightarrow$$
$$\sum_{g\in P}v_g^{g\max}GMAX_g=\sum_n\left(\sum_d D_{d,n}DQTY_d\right)u_n-\sum_l v_l^{t\max}TCAP_l+$$
$$-\sum_l v_l^{t\min}TCAP_l+\sum_g\sum_n G_{g,n}MC_gGQTY_g-\sum_{g\notin P}v_g^{g\max}GMAX_g$$

(5)

The right-hand side of equation (5) involves only linear terms. Now we show that the left-hand side is the profit function of portfolio p.