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Inventory control and investment policy

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Abstract

The idea underlying this paper is to consider inventory control as part of a wider class of economic problems, namely financial risk management. Only discrete-time models are investigated here. In the framework of the Cox–Ross–Rubinstein model of a financial market a combined inventory replenishment and investment policy is obtained. Company solvency is studied as well.

Keywords: Inventory control; Financial market; Cost approach; Reliability approach; Solvency

1. Introduction

The aim of this paper is to draw attention of inventory researchers to new perspectives opened by treating inventory problems within the wider scope of financial risk management.

In the year 2001 we celebrate the half century anniversary of the pioneering work of Arrow et al. (1951). Along with the seminal papers of Dvoretzky et al. (1952, 1953) it formed the foundation of modern inventory theory (or, more strictly, its cost approach). It is interesting to recall that the 1971 Nobel Prize winner, the Honorary President of ISIR Kenneth Arrow, has also greatly contributed to the development of financial economics.

The classical (AHM) inventory model and its various modifications (see, e.g. Chikán, 1986) usually took into account inventory replenishment and holding costs, as well as a shortage penalty. Although some authors (see, e.g., Rustenburg et al., 1999) considered budget restrictions, for the most part the cash amount available for inventories was assumed to be unlimited and the decisions were aimed at the minimization of expected accumulated costs (since demand was supposed to be random). There was a lot of discussion about capital tied up in inventories (see, e.g., Pujawan and Kingsman, 1999). We mention in passing that, in an attempt to avoid the undesirable effects of inventory holding, a new just-in-time concept has been introduced in inventory theory and practice (see, e.g., Groenevelt, 1993). One could also say that *implicitly* the existence of a financial market was taken into account from the outset by discounting future expenses.

Now the time has come to use explicitly and in full the powerful tools of financial mathematics by combining inventory control with investment policy.

For simplicity we consider below only discrete-time models. Section 2 contains some preliminary results. First we point out the effect of a budget constraint in the classical inventory model. Then

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we drop the nonnegativity restriction on inventory order, allowing for seasonal sales. The one-period demand is assumed to be a random variable with a finite mean (taking negative values as well in order to incorporate the possibility of customers returning bought product or remanufacturing).

In Section 3 we introduce the Cox–Ross–Rubinstein model of a financial market (for more details see the fundamental book by Shiryaev (1999)). Assuming an inventory replenishment policy to be fixed we establish the optimal investment strategy. Using a reliability approach we treat the company solvency problem. In the framework of the cost approach we obtain the optimal inventory replenishment and investment policy. Almost all proofs are omitted due to lack of space.

In Section 4 we draw conclusions and outline further research directions.

2. Preliminary results

To illustrate the influence of various constraints on optimal inventory control we consider the simplest case. The replenishment order, made at the beginning of a period, is delivered immediately and can be used to satisfy the arising demand. The inventory left is stored for future use and a penalty is paid for a stock-out.

Let the demand process be described by a sequence of independent identically distributed random variables $\{\xi_i\}_{i\geqslant 1}$. We suppose ξ_i to be nonnegative and to have a density $\varphi(s)>0,\ s>0$, as well as a finite mean. All the cost functions are assumed to be linear, their slopes being, respectively, c_1, the ordering cost per unit, h, the holding cost per unit per period, and p, the shortage penalty per unit per period. Unsatisfied demand is backlogged and hence x can be negative. We also introduce the discount factor $\alpha,\ 0<\alpha<1$.

Denote, as usual, by $f_n(x)$ the minimal expected n-period cost, x being the initial inventory level. Then we have the following dynamic programming equation for $n\geqslant 1$:

$$f_n(x) = -c_1 x + \min_{y\geqslant x} G_n(y), \tag{1}$$

with

$$G_n(y) = c_1 y + L(y) + \alpha \int_0^{\infty} f_{n-1}(y-s)\varphi(s)\,\mathrm{d}s \tag{2}$$

and

$$L(y) = h\int_0^{y} (y-s)\varphi(s)\,\mathrm{d}s + p\int_y^{\infty} (s-y)\varphi(s)\,\mathrm{d}s. \tag{3}$$

The y-value, say $y_n(x)$, minimizing the right-hand side of (1), is called the optimal ordering level. It is well known (see, e.g., Bulinskaya, 1990) that the following result is valid.

Theorem 1. *If $p>c_1$, then there exists an increasing sequence $\{y_n\}_{n\geqslant 1}$ such that*

$$y_n(x) = \begin{cases} y_n & \text{for } x\leqslant y_n,\\ x & \text{for } x>y_n. \end{cases}$$

Moreover, $\bar{y} = \lim_{n\to\infty} y_n$ is the solution of the following equation:

$$\int_0^{y} \varphi(s)\,\mathrm{d}s = (p - c_1 + \alpha c_1)(p+h)^{-1}.$$

As in Bulinskaya (1990) the following result can be proved.

Theorem 2. *The stationary inventory policy determined by a single critical level $\bar{y}$ is asymptotically optimal.*

(An inventory policy is said to be asymptotically optimal if the long-run average cost per period, under this policy, is equal to $\lim_{n\to\infty} n^{-1} f_n(x)$ with $f_n(x)$ defined by (1).)

Now let u be the cash amount available each period for inventory replenishment. Then the corresponding minimal costs $\bar{f}_n(x)$ satisfy, instead of (1), a similar relation with minimization carried out over the interval $[x, x+uc_1^{-1}]$.

Theorem 3. *Under the assumption of Theorem 1 the optimal ordering policy $\bar{y}_n(x)$ is determined by the*