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Spare parts inventory control considering stochastic growth of an installed base

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A B S T R A C T

Installed base is a measure describing the number of units of a particular system actually in use. To maintain the performance of the installed units, spare parts inventory control is extremely important and becomes very challenging when the installed base changes over time. This problem is often encountered when a manufacturer starts to deliver a new product to customers and agrees to provide spare parts to replace failed units in the future. To cope with the resulting non-stationary stochastic maintenance demand, a spare parts control strategy needs to be carefully developed. The goal is to ensure that timely replacements can be provided to customers while minimizing the overall cost for spare parts inventory control. This paper provides a model for the aggregate maintenance demand generated by a product whose installed base grows according to a homogeneous Poisson process. Under a special case where the product's failure time follows the exponential distribution, the closed form solutions for the mean and variance of the aggregate maintenance demand are obtained. Based on the model, a dynamic (Q, r) restocking policy is formulated and solved using a multi-resolution approach. Two numerical examples are provided to demonstrate the application of the proposed method in controlling spare parts inventory under a service level constraint. Simulation is utilized to explore the effectiveness of the multi-resolution approach.

1. Introduction

Spare parts inventory control is usually implemented to maximize the availability of a fleet of systems that require service throughout their life cycle. Comprehensive reviews on spare parts inventory control can be found in Kennedy, Patterson, and Fredenhall (2002), Nahmias (1981) and Zipkin (2005). Existing models can be classified into either a single-echelon inventory model (Cohen, Kleindorfer, Lee, & Pyke, 1992) or a multi-echelon model (Thonemann, Brown, & Hausman, 2002). Furthermore, according to various operating parameters, these models can be further classified into a fixed quantity model and a fixed period model. More specifically, two popular inventory control models have been widely used: (1) the lot-size/reorder point (Q, r) model (Hopp, Zhang, & Spearman, 1999), where Q represents the order quantity, and r is the reorder point; and (2) the reorder point/order-up-to-level (S, s) (Cohen, Kleindorfer, Lee, & Pyke, 1992), where S represents the order-up-to-level, and s is the reorder point. To determine the optimal operating parameters, associated maintenance demands need to be characterized first. In the literature, most inventory models (Grave, 1985; Gross, Miller, & Soland, 1985; Jung, 1993; Kupta and Rao, 1996; Slay, Bachman, Kline, O'Malley, Eichorn, & King, 1996) are derived based on one of the following assumptions: (1) maintenance demands follow a homogeneous Poisson process (constant rate); (2) installed base is fixed, which generates stationary maintenance demands. In other words, those assumptions address inventory control problems when the distribution of maintenance demand does not change over time. Such distribution is usually determined by fitting an assumed distribution to historical demand data or by simply utilizing past experience.

In practice, those assumptions may not be realistic, especially for a new product, for which the maintenance demands grow rapidly as the field installations increase. In the literature, however, little effort has been given to spare parts inventory control considering stochastic growth of an installed base. Jin, Liao, Xiong, and Sung, (2006) showed that stationary maintenance demand models underestimate the actual maintenance demands in such cases. As a result, it is important to investigate how the product population grows in the field in order to proactively forecast the upcoming maintenance demands for dynamic control of spare parts inventory. This paper addresses a spare parts inventory control problem for a non-repairable product with a stochastically growing installed base. When an installed unit fails it will be replaced by a new one. The closed form solution for the maintenance demand is derived when new sales occur following a homogenous Poisson process and the failure time of an installed unit follows the exponential distribution. Based on the model for the maintenance demand, a (Q, r) inventory control model is formulated, and the

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operating parameters are optimized using a multi-resolution technique.

The remainder of this paper is organized as follows. In Section 2, the maintenance demand is derived for a product with a generally distributed lifetime. Section 3 addresses the special case where the product's lifetime follows the exponential distribution. Section 4 presents the formulation for the dynamic (Q, r) spare parts inventory problem and the multi-resolution solution technique. In Section 5, two numerical examples are provided to demonstrate the proposed approach in practical use. Simulation is utilized to investigate the effectiveness of the multi-resolution approach. Finally, conclusions are provided in Section 6.

2. Maintenance demand considering the growth of product installed base

Notations for modeling maintenance demand

$N(t)$	the number of new sales by time t, and $N(t)+1$ is the installed base
λ	rate of new sales or the growth rate of the unit
W_i	the arrival time of the ith new sale
a	constant failure rate when the product's failure time distribution is exponential
$F(t)$	cumulative distribution function (Cdf) of the product's lifetime
$F^{(i)}(t)$	i-fold convolution of $F(t)$
$H(t)$	the expected number of renewals or failure replacements by time t if a unit is installed at time 0
$S(t)$	aggregate maintenance demand at time t

The following assumptions are made regarding the product of our interest:

(1) New sales increase the installed base, which are fulfilled through a process independent of the spare parts procurement and control process. After the first unit is installed at time $t=0$, new sales occur according to a homogeneous Poisson process with the rate of λ (i.e., λ installations per unit time). The total number of new sales $N(t)$ by time t has the probability mass function:

$$\Pr\{N(t)=n\}=\frac{(\lambda t)^n e^{-\lambda t}}{n!}, \quad \text{for } n=0,1,2,\ldots \tag{1}$$

Note that the installed base is equal to $n+1$ when the number of additional sales is $N(t)=n$ by time t.

(2) The product is non-repairable, and an installed unit will be replaced by a new one upon failure.

(3) Downtime due to replacement (including disassembly, installation, and waiting for a spare part) can be ignored as compared to the lifetime of the product.

2.1. Renewal processes generated after new sales

Fig. 1 illustrates occurrences of new sales together with the subsequent maintenance demands. Specifically, the first unit is installed at time $t=0$. After a random period of time W_1, a new sale occurs and the second unit is installed, which may or may not be at the same customer site. We will use a homogeneous Poisson process to model the occurrence of a new sale. On the other hand, each unit once installed will fail after a random period of time and be replaced by a new one upon failure. The counting process associated with each installed unit belongs to a renewal process. Consequently, if n new sales have occurred by time t, there will be $n+1$ renewal processes with most likely different starting points in time.

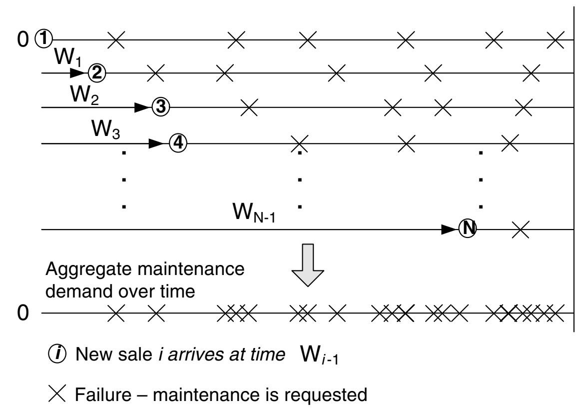

Fig. 1. Maintenance demand vs. new sales.

Let $F(t)$ be the cumulative distribution function (Cdf) of the product's lifetime T. For the renewal process generated by the first installed unit, the expected number of renewals (failure replacements) up to time t can be expressed as:

$$H(t)=F(t)+\int_0^t H(t-\tau)\mathrm{d}F(\tau)=\sum_{i=1}^{\infty}F^{(i)}(t) \tag{2}$$

This is the well-known renewal integral equation, where $F^{(i)}(t)$ is the i-fold convolution of the underlying failure time distribution $F(t)$ recursively defined by

$$\begin{cases} F^{(i)}(t)=\int_0^t F^{(i-1)}(t-\tau)\mathrm{d}F(\tau), & t\geqslant\tau\geqslant 0, \quad i\geqslant 2 \\ F^{(1)}(t)=F(t), & t\geqslant 0 \end{cases} \tag{3}$$

Eq. (2) has a closed form solution only for a few failure time distributions. For example, if $F(t)$ is exponentially distributed with a constant failure rate a, then the corresponding solution is $H(t)=at$. For many other distributions such as Weibull, normal and log-normal distributions, the analytical solutions for $H(t)$ may not be obtained, thus have to be solved numerically. In practice, the direct Riemann–Stieltjes integration approach proposed by Xie (1989) may be utilized. For the case involving the Weibull distribution, the analytical approximation can be used based on the recent work done by Jiang (2008).

It is worth emphasizing that Eq. (2) gives the expected number of renewals when the process starts from time $t=0$. The maintenance problem addressed in this paper actually involves multiple renewal processes, each of which has its own starting time W_k, for $k=0, 1, 2, \ldots$, i.e., the arrival time of the kth new sale (note: $W_0=0$ and $H_0(t)=H(t)$). As a result, the renewal function $H_k(t)$ for the kth renewal process (i.e., kth new sale) can be treated as $H(t)$ with a time delay W_k

$$H_k(t)=H(t-W_k), \qquad W_k<t, \quad \text{for } k=1,2,\ldots \tag{4}$$

Since new sales occur according to a homogeneous Poisson process, W_k is a random variable, consequently, so is the renewal function $H_k(t)$.

2.2. Generic aggregate maintenance demand

When dealing with the spare parts inventory control, the aggregate maintenance demand, i.e., the total number of renewals, is of our interest. For instance, as shown in Fig. 1, the customer with the third installation has experienced four renewals (i.e., four failures that consumed four spare units) by time t. Similarly, the customer with the fourth installation has experienced three renewals by time t. In calculating the aggregate maintenance demand by time t, the numbers of renewals generated by all the installed units will