Available online at www.sciencedirect.com
SCIENCE DIRECT
Electric Power Systems Research 73 (2005) 353–362
ELECTRIC POWER SYSTEMS RESEARCH
www.elsevier.com/locate/epsr

# New control approach for four-wire active power filter based on the use of synchronous reference frame

M.C. Benhabib*, S. Saadate

*Groupe de Recherche en Electrotechnique et Electronique de Nancy, GREEN-UHP CNRS UMR 7037, Université Henri Poincaré Nancy 1, B.P. 239, 54506 Vandoeuvre lès Nancy, France*

Received 21 June 2004; received in revised form 9 August 2004; accepted 17 August 2004
Available online 26 November 2004

**Abstract**

Since the development of the first theories for the active power filters (APF) control, many efforts have been concentrated to improve their performances because the number of non-linear loads did not stop to increase. Moreover, the appearance of non-linear load of different types, three-phase or single-phase, provokes unbalanced system, which requires more elaborated controls especially in term of robustness. For the unbalanced non-linear loads, the reduction of neutral current is one of the two aims of different APF's controls studied in this paper. This will be highly appreciated by the distribution networks. The second aim is naturally to obtain an acceptable THD for each line current.

In this paper, three control methods, all based on the instantaneous powers theory introduced by H. Akagi [H. Akagi, Y. Kanazawa, A. Nabae, Generalized theory of the instantaneous reactive power in three-phase circuits, in: Proceedings of International Power Electronics Conference, Tokyo, Japan (1983) 1375–1386.] are compared and advantages and drawbacks of each approach are discussed. Then, the improvements will be presented for these controls so as to obtain better results in case of unbalanced system. The simulation results will show the effectiveness of these improvements. Finally, in order to improve again the performances of APF, a new control is proposed. This approach is based on the use of synchronous reference frame and its effectiveness will be validated through numerical simulations.


*Keywords:* Active power filter (APF); Cross-vector, *p–q* and *p–q–r* control theory; Synchronous reference frame

## 1. Introduction

In the last two decades an unceasingly development of active power filters (APF's) control has been performed. The most important concerns the control methods based on the instantaneous powers introduced by Akagi [1]. Many other control strategies to obtain good filtering results for four-wire electrical networks have been also proposed [8–11].

In the first part of this paper, three classical APF controls which are *p–q* theory [7], cross-vector control [2,3] and *p–q–r* theory [4,6] will be presented. The THD's of the lines current and the rms value of the neutral current are the quality criteria chosen all over this paper. Then, improvements for these controls to minimise the influence of unbalanced and eventually voltage harmonics on the filtering and neutral current results are suggested. Finally, synchronous reference frame is introduced in improved cross-vector control method. Among three approaches, this one has been chosen as it uses fewer operations for practical realisation. The new approach is then applied to the control of a three-phase four-wire APF and its effectiveness is validated through simulations.

All simulations are performed using Matlab—Simulink Power Blockset ToolBox. They are realised in the same conditions, with the same parameters for the system and control so as the obtained results can be compared with each other.

## 2. Basic control theories for APF

In this section, three control methods are briefly presented and then the simulation results are discussed.

* Corresponding author.
*E-mail addresses:* mc.benhabib@green.uhp-nancy.fr (M.C. Benhabib), shahrokh.Saadate@green.uhp-nancy.fr (S. Saadate).

0378-7796/$ – see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.epsr.2004.08.012

### 2.1. p–q–o theory

This theory introduced in 1983 by Akagi et al. [1,2] has been developed by the authors to be used in a four-wire three-phase system. This theory could be presented as follows.

Consider a three-phase four-wire system including zero-sequence voltage and zero-sequence current. The three-phase voltages and currents in *a–b–c* co-ordinates are transformed to *α–β–*0 co-ordinates by the following equations:

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (2)$$

The zero-sequence instantaneous real power $p_0$, the *α–β* instantaneous real and imaginary power, *p* and *q*, are defined as follows:

$$\begin{bmatrix} p \\ q \\ p_0 \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta & 0 \\ -v_\beta & v_\alpha & 0 \\ 0 & 0 & v_0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \quad (3)$$

The instantaneous power *p* can be decomposed to $p = \overline{p} + \tilde{p}$, with $\overline{p}$ the continuous component and $\tilde{p}$ the harmonic component.

In order to extract only the current harmonics, which will be injected by the APF, the continuous component, $\overline{p}$, should be eliminated. Thus, the currents in *α–β–*0 co-ordinates will become:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \frac{1}{v_0 v_\alpha^2 + v_0 v_\beta^2} \begin{bmatrix} v_0 v_\alpha & -v_0 v_\beta & 0 \\ v_0 v_\beta & v_0 v_\alpha & 0 \\ 0 & 0 & v_\alpha^2 + v_\beta^2 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ q \\ p_0 \end{bmatrix} \quad (4)$$

By retransforming from *α–β–*0 co-ordinates to *a–b–c* co-ordinates, named in this case $i_{\text{a–h}}$, $i_{\text{b–h}}$, $i_{\text{c–h}}$, one obtains:

$$\begin{bmatrix} i_{a\text{–}h} \\ i_{b\text{–}h} \\ i_{c\text{–}h} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \quad (5)$$

and the neutral current is equal to:

$$i_{\text{n}} = (i_{a\text{–}h} + i_{b\text{–}h} + i_{c\text{–}h})$$

### 2.2. The cross-vector theory

The cross-vector theory is a modified *p–q* theory, and differs from only in the mapping matrices [3]. The instantaneous powers will be defined as follows:

$$\begin{bmatrix} p \\ q_0 \\ q_\alpha \\ q_\beta \end{bmatrix} = \begin{bmatrix} v_0 & v_\alpha & v_\beta \\ 0 & -v_\beta & v_\alpha \\ v_\beta & 0 & -v_0 \\ -v_\alpha & v_0 & 0 \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \quad (6)$$

As in the *p–q* theory, it is possible to divide the instantaneous active power into two parts $p = \overline{p} + \tilde{p}$. By filtering the continuous component, one obtains the currents in *α–β–*0 co-ordinates as follows:

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{v_0^2 + v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_0 & 0 & v_\beta & -v_\alpha \\ v_\alpha & -v_\beta & 0 & v_0 \\ v_\beta & v_\alpha & -v_0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ q_0 \\ q_\alpha \\ q_\beta \end{bmatrix} \quad (7)$$

Once again the currents in *a–b–c* co-ordinates are expressed by:

$$\begin{bmatrix} i_{a\text{–}h} \\ i_{b\text{–}h} \\ i_{c\text{–}h} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \quad (8)$$

The neutral current is determined as before:

$$i_{\text{n}} = i_{a\text{–}h} + i_{b\text{–}h} + i_{c\text{–}h}$$

### 2.3. p–q–r theory

The *p–q–r* theory proposed by Kim et al. [4] can be defined as follows.

After a transformation of currents and voltages from *a–b–c* co-ordinates to *α–β–*0 co-ordinates, according to Eqs. (1) and (2), the currents are transformed from *α–β–*0 co-ordinates to *p–q–r* co-ordinates as follows:

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} = \begin{bmatrix} \frac{v_\alpha}{v_{\alpha\beta0}} & \frac{v_\beta}{v_{\alpha\beta0}} & \frac{v_0}{v_{\alpha\beta0}} \\ -\frac{v_\beta}{v_{\alpha\beta0}} & \frac{v_\alpha}{v_{\alpha\beta}} & 0 \\ -\frac{v_\alpha v_0}{v_{\alpha\beta}\, v_{\alpha\beta0}} & -\frac{v_\alpha v_\beta}{v_{\alpha\beta}\, v_{\alpha\beta0}} & \frac{v_{\alpha\beta}^2}{v_{\alpha\beta}\, v_{\alpha\beta0}} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \quad (9)$$

where $v_{\alpha\beta} = \sqrt{v_\alpha^2 + v_\beta^2}$ and $v_{\alpha\beta0} = \sqrt{v_\alpha^2 + v_\beta^2 + v_0^2}$.

According to the authors, the instantaneous active and reactive power will be defined by:

$$\begin{bmatrix} p \\ q_r \\ q_q \end{bmatrix} = v_{\alpha\beta0} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} \quad (10)$$

As for the *p–q* theory, it is also possible to separate the instantaneous active power into two parts, $p = \overline{p} + \tilde{p}$, and