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Inventory control as a discrete system control for the fixed-order quantity system

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ABSTRACT

This paper shows how to model a problem to find optimal number of replenishments in the fixed-order quantity system as a basic problem of optimal control of the discrete system. The decision environment is deterministic and the time horizon is finite. A discrete system consists of the law of dynamics, control domain and performance criterion. It is primarily a simulation model of the inventory dynamics, but the performance criterion enables various order strategies to be compared. The dynamics of state variables depends on the inflow and outflow rates. This paper explicitly defines flow regulators for the four patterns of the inventory: discrete inflow – continuous/discrete outflow and continuous inflow – continuous/discrete outflow. It has been discussed how to use suggested model for variants of the fixed-order quantity system as the scenarios of the model. To find the optimal process, the simulation-based optimization is used.

1. Introduction

As you can see in Axsäter [1], Russell and Taylor [2], Vollmann et al. [3], Chase and Aquilano [4], Barlow [5], Muller [6], Wild [7], even new books dealing with inventory control, describe a classical economic order quantity model and its variants when demand rate is constant and known, as a starting point for further understanding of inventory dynamics.

Economic order quantity (also known as the Wilson EOQ Model or simply the EOQ Model) is a model that defines the optimal quantity to order that minimizes total variable costs required to order and hold inventory. The model was originally developed by Harris [8], though Wilson [9] is credited for his early in-depth analysis of the model. It was a time without easy affordable computers and the simple useful mathematical models was preferred; see Erlencotter [10] for its history.

A discrete time system is a more natural manner to describe inventory dynamics. Model of discrete system control is both a simulation model of inventory dynamics and an optimization model which can give optimal control according to the defined performance criterion.

There are numerous articles using the discrete time system in the deterministic inventory control. These articles address mainly the lot-sizing problems, beginning with Wagner and Whitin [11], Scarf [12]. In order to find an optimal inventory control for various variants of the dynamic lot-sizing problems, dynamic programming algorithms [13] can be applied. Setchi and Thompson [14] have shown how to apply optimal control theory to management science. There are numerous meta-heuristics algorithms for dynamic lot-sizing problems; see Zoller and Robrade [15] and Jans and Degraeve [16] for an overview.

The classical EOQ model is considered as a continuous-time approach whereas the lot-sizing problem is considered as a discrete-time approach. Khmelnitsky and Tzur [17] have analyzed a parallelism of continuous-time and discrete-time

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production planning problems. They considered the lot-sizing problem "as the discrete counterpart of the EOQ, since it is merely a special case of the dynamic demand case".

The classical EOQ model tackles explicitly the fixed-order quantity system. Lot-sizing models address mainly the periodic review systems. EOQ model anticipates the saw-tooth pattern as the inventory dynamics. Articles, using dynamic programming approach [13], model the dynamics of the stock X_t by using an inflow variable u_t as unknown, and outflow variable w_t as deterministic or stochastic one ($X_{t+1} = X_t + u_t - w_t$, $t = 1,2,\ldots,T$). Both of them subject the model to the optimization method used.

In the simulation-based optimization [18] there is a complete separation between the model that represents the system and the procedure that is used to solve optimization problems defined within this model. The simulation model can change and evolve to incorporate additional elements, while the optimization routines remain the same; see Swisher and Hyden [19] and Fu et al. [20] for an overview.

The theoretical foundations for the model of the optimal control of discrete systems can be found in the work of Boltianski [21]. I will interpret it with some additional information.

The principal variable in this approach is discrete time t taking integer values $t = 0,1,2,\ldots,T$, where T is the number of days of time horizon.

The state of a system is represented by values of N variables X (X^n, $n = 1,2,\ldots,N$) called "state variables". For each variable X^n $(n = 1,2,\ldots,N)$ there is a co-ordinate of E^N Euclid's space called "state space". The state of moving object in time t can be represented as the point of N-dimensional "state space".

Variables, affecting the dynamics of the system and whose values are fixed and known in advance, will be called "circumstances variables". There will be S circumstances variables p, denoted as (p^s, $s = 1,2,\ldots,S$). For each variable p^s $(s = 1,2,\ldots,S)$ there is a co-ordinate of E^S Euclid's space called "circumstances space". The circumstances of the moving object in time t can be represented as a point of the S-dimensional "circumstances space".

Finally, there will be R variables whose values are unknown and are to be found according to some criteria. These variables will be called "control variables" and denoted as u^r $(r = 1,2,\ldots,R)$. For each variable u^r $(r = 1,2,\ldots,R)$ there is a co-ordinate of E^R Euclid's space called "control space". A control in time t can be represented as the point of R-dimensional "control space".

Further in the text, superscripts will be used as labels of co-ordinates of appropriate spaces. Subscripts will denote time at which a variable takes a value (current time t or previous time $t-1$).

Interrelation among all selected variables can be represented by the law of dynamics:

$$\begin{aligned} &X_0 = \text{known}, \\ &X_t = f(X_{t-1}, p_t, u_t), \quad t = 1,2,\ldots,T \end{aligned} \tag{1}$$

where

X_0 known initial state of the system
f: $(f^1,f^2,\ldots,f^N)$ N-dimensional vector function with values in E^N space
X_t value of N-dimensional vector function at current time t
X_{t-1} value of N-dimensional vector function at previous time $t-1$
p_t value of S-dimensional vector at current time t
u_t value of R-dimensional vector at current time t

A state X_t is obtained as a value of the vector function $f(X_{t-1},p_t,u_t)$ based on the state X_{t-1} from the previous time $t-1$ and values of the circumstances variables p_t and control variables u_t from the current time t.

An un-empty set $U_t \in (X_{t-1},p_t)$ in the space of variables $u^1,u^2,\ldots,u^R$ is to be determined for each state point $X \in E^N$ and each $t = 1,2,\ldots,T$. It is called control domain depending on the value of the state variable X_{t-1} from the previous time $t-1$ and circumstances variable from the current time t. The control u_t can take a value merely from the control domain

$$u_t \in U_t(X_{t-1}, p_t), \quad t = 1,2,\ldots,T. \tag{2}$$

where subscripts denote time at which a variable takes value.

Relations of the law of dynamics (1) and control domain (2) determine a discrete controlled object. These relations also represent the simulation model of the moving object.

A series of the circumstances points throughout time horizon $p_1,p_2,\ldots,p_T$ is called circumstances of the moving object. A series of the control points throughout time horizon $u_1,u_2,\ldots,u_T$ is called control of the moving object. A series of the state points throughout time horizon $X_1,X_2,\ldots,X_T$ is called trajectory of the moving object. A logical troika (X,p,u) consisted of trajectory (X), circumstances (p) and control (u) is called discrete process.

The success of the control will be measured at each time t ($t = 1,2,\ldots,T$) by defined function $f^0(X_{t-1},p_t,u_t)$. The performance criterion J is an objective functional that adds values of function f^0 throughout time horizon, i.e.

$$J = f^0(X_0,p_1,u_1) + f^0(X_1,p_2,u_2) + \cdots + f^0(X_{t-1},p_t,u_t) = \sum_{t=1}^{T} f^0(X_{t-1},p_t,u_t). \tag{3}$$