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A third order model for the doubly-fed induction machine

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Abstract

Induction generators are generally simulated by means of a well-known model described by Brereton et al. [1], based on the induction motor equations derived by Stanley [2]. In this model the possibility of opening the rotor circuit in order to inject a voltage source is not taken into account, although there are other models where it is dealt with [3]. This paper presents an alternative way of obtaining the mentioned model and introduces the possibility of modeling voltage sources in the rotor circuit, which can be very useful when simulating some generating schemes, such as variable speed asynchronous wind turbines.

Keywords: Induction machine models; Stator; Rotor

1. Introduction

Induction generators are the most commonly used electric generators in industry, and they are particularly common in some renewable energy applications such as wind turbines (WTs). Several models have been proposed to simulate them. The model proposed by Brereton et al. is of great use when induction generators must be dynamically simulated. This model is a very valid one when the machine works with its rotor short-circuited, that is, the rotor voltage having a value of 0. The trend for future years seems to be the use of variable-speed WTs, in order to achieve an optimization of the generated power profiles. Several solutions have been proposed, such as the use of synchronous generators linked to the electrical network through electronic devices, and doubly-fed induction generators, which this paper is concerned with. The simulation of doubly-fed induction generators has been dealt with in many papers, and generally involves handling a set of differential equations related to the rotor and stator currents, fluxes and voltages. The purpose of this paper is to obtain a set of more simplified equations than those generally used.

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2. The third order models of the induction machine

First, the matrices k_s and k_r [4,5] will be assumed, which allows one to formulate a change of variables transforming them from the three-phase variables system to an arbitrary reference frame. In this case the new reference frame is rotating at synchronous speed. Both matrices can be seen in Appendix A.

The following notation will be employed:

- I_{r123} is a vector whose components are the per phase rotor currents ($I^T_{r123} = (I_{r1}I_{r2}I_{r3})$), and I_{rqd0} ($I^T_{rqd0} = (I_{rq}I_{rd}I_{r0})$) is a vector whose components are the rotor currents seen from the $dq0$ axes. The superindex T is used for notating transpose matrices.
- I_{sabc} and I_{sqd0} are the stator currents in both reference frames.
- V_{r123} and V_{rqd0} are the rotor voltages.
- V_{sabc} and V_{sqd0} are the stator voltages.
- φ_{r123} and φ_{rqd0} are the rotor fluxes.
- φ_{sabc} and φ_{sqd0} are the stator fluxes.

The following equations express the relationship between both reference frames:

$$V_{rqd0} = k_r V_{r123} \Rightarrow V_{r123} = k_r^{-1} V_{rqd0} \tag{1a}$$

$$V_{sqd0} = k_s V_{sabc} \Rightarrow V_{sabc} = k_s^{-1} V_{sqd0} \tag{1b}$$

$$I_{rqd0} = k_r I_{r123} \Rightarrow I_{r123} = k_r^{-1} I_{rqd0} \tag{1c}$$

$$I_{sqd0} = k_s I_{sabc} \Rightarrow I_{sabc} = k_s^{-1} I_{sqd0} \tag{1d}$$

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$$\varphi_{rqd0} = k_r \varphi_{r123} \Rightarrow \varphi_{r123} = k_r^{-1} \varphi_{rqd0} \tag{1e}$$

$$\varphi_{sqd0} = k_s \varphi_{sabc} \Rightarrow \varphi_{sabc} = k_s^{-1} \varphi_{sqd0} \tag{1f}$$

where the matrices k_r and k_s are given in Appendix A, where both reference frames can also be seen.

In order to obtain the model, the following equations, valid for the rotor and stator electrical circuits, must be taken into account:

$$\begin{pmatrix} V_{r123} \\ V_{sabc} \end{pmatrix} = \begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} I_{r123} \\ I_{sabc} \end{pmatrix} + \frac{d}{dt} \begin{pmatrix} \varphi_{r123} \\ \varphi_{sabc} \end{pmatrix} \tag{2}$$

where

$$r = \begin{bmatrix} R_R & 0 & 0 \\ 0 & R_R & 0 \\ 0 & 0 & R_R \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix},$$

R_R is the rotor resistance and R_s the stator resistance. The machine is assumed to be balanced. Taking this assumption into account, the component '0' can be neglected. So, from now on only the 'd' and 'q' components are employed.

Changing the reference frame by means of the matrices

$$\begin{pmatrix} k_r & 0 \\ 0 & k_s \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} k_r & 0 \\ 0 & k_s \end{pmatrix}^{-1},$$

the following is obtained:

$$\begin{pmatrix} V_{rqd} \\ V_{sqd} \end{pmatrix} = \begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} I_{rqd} \\ I_{sqd} \end{pmatrix} + \begin{pmatrix} k_r & 0 \\ 0 & k_s \end{pmatrix} \frac{d}{dt} \left(\begin{pmatrix} k_r^{-1} & 0 \\ 0 & k_s^{-1} \end{pmatrix} \begin{pmatrix} \varphi_{rqd} \\ \varphi_{sqd} \end{pmatrix} \right) \tag{3}$$

2.1. Model for the induction machine with short-circuited rotor

In the case of an induction generator with short-circuited rotor, which is a common solution in constant speed WTs , the previous equations become the following, by neglecting the stator transients $d\varphi_{sqd}/dt = 0$:

$$\begin{pmatrix} 0 \\ V_{sqd} \end{pmatrix} = \begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} I_{rqd} \\ I_{sqd} \end{pmatrix} + \begin{bmatrix} k_r \dfrac{dk_r^{-1}}{dt} & 0 \\ 0 & k_s \dfrac{dk_s^{-1}}{dt} \end{bmatrix} \begin{pmatrix} \varphi_{rqd} \\ \varphi_{sqd} \end{pmatrix} + \begin{bmatrix} \dfrac{d\varphi_{rqd}}{dt} \\ 0 \end{bmatrix} \tag{4}$$

The rotor and stator fluxes can be written as functions of their currents, as follows:

$$\begin{pmatrix} \varphi_{rqd} \\ \varphi_{sqd} \end{pmatrix} = \begin{pmatrix} L_{rr} & L_{rs} \\ L_{rs} & L_{ss} \end{pmatrix} \begin{pmatrix} I_{rqd} \\ I_{sqd} \end{pmatrix} \tag{5}$$

where:

$$L_{rr} = \begin{pmatrix} l_{rr} & 0 \\ 0 & l_{rr} \end{pmatrix} \tag{6a}$$

$$L_{ss} = \begin{pmatrix} l_{ss} & 0 \\ 0 & l_{ss} \end{pmatrix} \tag{6b}$$

$$L_{rs} = \begin{pmatrix} l_{rs} & 0 \\ 0 & l_{rs} \end{pmatrix} \tag{6c}$$

and:

- $l_{rr} = l_2 + l_{rs}$, and l_2 is the leakage inductance in a phase of the rotor.
- $l_{ss} = l_1 + l_{rs}$, and l_1 is the stator leakage inductance.
- l_{rs} is the magnetizing inductance.

From Eq. (5) the following expression for the rotor currents can be obtained:

$$I_{rqd} = L_{rr}^{-1}(\varphi_{rqd} - L_{rs} I_{sqd}) \tag{7}$$

Now the rotor short-circuit constant is defined as $T_0' = l_{rr}/r$. So from Eq. (4) the following equation can be obtained:

$$\frac{d\varphi_{rqd}}{dt} = -k_r \frac{dk_r^{-1}}{dt} \varphi_{rqd} - \frac{1}{T_0'}(\varphi_{rqd} - L_{rs} I_{sqd}) \tag{8}$$

On the other hand, from Eqs. (5) and (7) the stator flux can also be written as:

$$\begin{aligned} \varphi_{sqd} &= L_{rs} I_{rqd} + L_{ss} I_{sqd} \\ &= L_{rs} L_{rr}^{-1} \varphi_{rqd} + (L_{ss} - L_{rs} L_{rr}^{-1} L_{rs}) I_{sqd} \end{aligned} \tag{9}$$

and also from Eqs. (4) and (9) the next equation for the stator voltage is valid:

$$\begin{aligned} V_{sqd} &= R I_{sqd} \\ &+ k_s \frac{dk_s^{-1}}{dt} (L_{rs} L_{rr}^{-1} \varphi_{rqd} + (L_{ss} - L_{rs} L_{rr}^{-1} L_{rs}) I_{sqd}) \end{aligned} \tag{10}$$