Bistability and Switching Properties of Semiconductor Ring Lasers With External Optical Injection

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***Abstract*—We investigate both analytically and numerically the switching, locking and stability properties of a bistable semiconductor ring laser subject to an external optical injection. Minimum optical power required for the injected signal at certain frequency to switch the lasing direction of a bistable semiconductor ring laser from its initially lasing direction to initially nonlasing direction is determined. Locking to the injected signal and stability of the switched laser are investigated to give an area of reliable switching operation. Correspondingly, numerical simulation has been carried out to find successful switching and stable locking region with variable injection power and frequency, and is compared with the analytical results. The region obtained from simulation coincides well with the intersection of switching, locking and stable locking regions. The relation between switching speed and parameters of injected source is also studied numerically.**

***Index Terms*—Bistability, injection locking, mode competition, semiconductor ring laser, switching.**

I. Introduction

SEMICONDUCTOR ring lasers (SRLs) have gained more and more attention owing to their unique feature of directional bistablility [1], [2], i.e., their ability to operate in two distinctive stable lasing directions (the clockwise (CW) and the counter-clockwise (CCW) directions) as shown in Fig. 1, and their potential applications in photonic systems for all-optical logic, optical switch, and optical memory applications [3], [4]. Switching from one lasing direction to the other can be triggered by externally injecting optical energy into the previously nonlasing direction, which starts a mode competition process favoring this direction for it to become the lasing direction at the end of the process.

Although the possibility and realization of achieving bistability in two mode lasers through mode competition by an external optical injection has been not only elegantly demonstrated mathematically using an idealized model but also experimentally [5]–[7], there are many questions about the switching process that need to be answered for any practical application. For the external optical injection source used to trigger the switching, relevant unknown aspects include the optical power or energy needed to induce switching, allowed detuning frequency range in which not only switching will happen, but the laser will stay stably locked to the external injection at the final state. In this paper, we extend the general theory to a more realistic and rigorous one for an SRL with two counter-propagating modes, and study the required conditions for realizing reliable switching in a SRL by an external optical signal.

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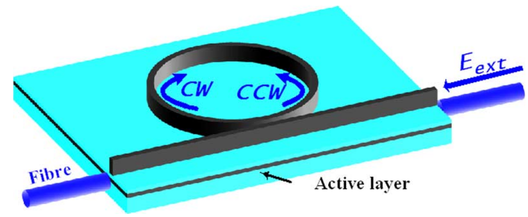

Fig. 1. Circular SRL with an external optical injection. The laser operates in the CCW direction before the optical injection ($E_{\rm ext}$) is added. It is switched to the CW direction after the injection.

The formula for the conditions needed for an externally injected continuous optical wave with certain power and detuning frequency to the free running frequency of the SRL is derived first. Locking and stability analysis is then carried out supposing that lasing direction has been switched to be the same as the external injection source. Results from numerical simulation are compared with the one obtained from theoretical study later. Detuning frequency and injection power's effects on switching speed are studied finally.

II. Theoretical Studies

A. Two-Mode Model

A two-mode model that analyzes the competitions between a pair of counter-propagating longitudinal modes at a single cavity resonance has successfully described SRLs as gyroscopes [8] and explained the observed alternating oscillation regime in the L–I characteristics of the SRL [9]. Our model is based on the basic two-mode model but includes an external injection term to account for the injected optical signal.

In single longitudinal mode operation, the electric field inside the ring cavity can be expressed as

$$E(x,t)=E_1(t)\exp[-i(\omega_{o1}t-kx)]+E_2(t)\exp[-i(\omega_{o2}t+kx)] \quad (1)$$

where E_1 and E_2 are the mean-field slowly varying complex amplitudes of the electric field associated with the two propagation direction modes, i.e., mode 1 is CCW and mode 2 is CW; x is the longitudinal spatial coordinate along the ring circumference, assumed positive in the CCW direction, and $\omega_{o1,2}$ is the optical frequency of the lasing longitudinal modes as the two directional modes should have a small detune between them [10].

The electric field of the external optical injection adding to the CW direction can be written as

$$E_{\rm ext}(x,t)=E_{\rm ext}(t)\exp\left[-i\left((\omega_{o2}t+\Delta\omega_{\rm ext}t+\Delta\phi_{\rm ext})+kx\right)\right] \tag{2}$$

where $\Delta\omega_{\rm ext}$ is the angular frequency detuning between the external injection and the free-running angular frequency ω_{o2} of the ring laser mode, and $\Delta\phi_{\rm ext}$ is the phase difference between them. The complex amplitudes of the electric fields are normalized so that $|E|^2$ equals to the density of photons. The time evolution of the fields in the cavity can be described by the following set of rate equations [9], [11], [12]:

$$\begin{aligned}\frac{dE_1}{dt}&=\frac{1}{2}(1-i\alpha)\left(\Gamma v_g g_n(N-N_{\rm tr})\right.\\&\qquad\left.\times\left(1-\varepsilon_s|E_1|^2-\varepsilon_c|E_2|^2\right)-\frac{1}{\tau_{p1}}\right)E_1\\&\quad+i(\omega_{o1}-\omega_{th})E_1\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{dE_2}{dt}&=\frac{1}{2}(1-i\alpha)\left(\Gamma v_g g_n(N-N_{\rm tr})\right.\\&\qquad\left.\times\left(1-\varepsilon_s|E_2|^2-\varepsilon_c|E_1|^2\right)-\frac{1}{\tau_{p2}}\right)E_2\\&\quad+i(\omega_{o2}-\omega_{th})E_2\\&\quad+K_{\rm ext}E_{\rm ext}\exp\left[-i(\Delta\omega_{\rm ext}t+\Delta\phi_{\rm ext})\right]\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{dN}{dt}&=\frac{\eta_i I}{eV}-\frac{N}{\tau_e}-v_g g_n(N-N_{\rm tr})\\&\quad\cdot\left(\left(1-\varepsilon_s|E_1|^2-\varepsilon_c|E_2|^2\right)|E_1|^2\right.\\&\qquad\left.+\left(1-\varepsilon_s|E_2|^2-\varepsilon_c|E_1|^2\right)|E_2|^2\right)\end{aligned} \tag{5}$$

where v_g is the group velocity, g_n is the differential gain at transparency, $N_{\rm tr}$ is the carrier density at transparency, ε_s and ε_c are the self-and cross-gain saturation coefficients respectively. α is the linewidth enhancement factor accounting for phase-amplitude coupling in the semiconductor medium, Γ represents the optical confinement factor which gives the spatial overlap between the active gain volume and the optical mode volume, $\tau_{p1,2}$ is the photon lifetime in the ring cavity for mode 1, 2 respectively, ω_{th} is the longitudinal resonant frequency at threshold. $K_{\rm ext}$ is the coupling parameter of the external injection. $E_{\rm ext}$ gives the field amplitude of injection. $\Delta\phi_{\rm ext}$ is always set to be 0, because the external field is always injected into the nonlasing direction which has little power initially. N is the carrier density in the active region. I is the bias current and η_i is the injection efficiency, e is the electronic charge. V is the volume for the quantum well active region, and τ_e is the carrier lifetime.

Linear coupling between the counter-propagating modes is caused by internal backscattering and external feedback [13] from the output waveguide facets, and can dominate SRL's operation at low bias current [9]. It is neglected because we are interested in the laser being biased high above threshold where nonlinear coupling is the fundamental physical mechanism that plays a much more important role as described in the (3) and (4) This is justifiable as relevant study [13] shows that only reflections with long time delay are a major concern. For short delay time (a few picoseconds as from integrated output waveguide facet), there is no feedback induced dynamics and the operating regimes mentioned in [9] can be reproduced. The reflection would only visibly reduce the extinction ratio between the lasing and nonlasing directions if facet reflection exceeds 10^{-3}, a level that is easily achievable with antireflection techniques such as coating.

Spontaneous emission noise is also neglected considering the SRL is biased high above the threshold current, and the operating point is chosen to be in the middle of a robust unidirectional region [9]. However, it may be important for the points at boundaries of a stable unidirectional region.

Equation (5) holds for the uniform carrier density N when any standing-wave pattern has a spatial period much smaller than carrier diffusion length and longitudinal variations of the carrier density are neglected.

Separating the magnitude part and phase part of the two-normalized-field (3)–(5) with $E(t)=A(t)e^{-j\phi}$, we have

$$\frac{dA_1}{dt}=\frac{1}{2}\left(\Gamma G_n(N-N_{\rm tr})\left(1-\varepsilon_s A_1^2-\varepsilon_c A_2^2\right)-\frac{1}{\tau_{p1}}\right)A_1 \tag{6}$$

$$\begin{aligned}\frac{d\phi_1}{dt}&=\frac{\alpha}{2}\left(\Gamma G_n(N-N_{\rm tr})\left(1-\varepsilon_s A_1^2-\varepsilon_c A_2^2\right)-\frac{1}{\tau_{p1}}\right)\\&\quad-(\omega_{o1}-\omega_{th})\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{dA_2}{dt}&=\frac{1}{2}\left(\Gamma G_n(N-N_{\rm tr})\left(1-\varepsilon_s A_2^2-\varepsilon_c A_1^2\right)-\frac{1}{\tau_{p2}}\right)A_2\\&\quad+K_{\rm ext}A_{\rm ext}\cos\phi_2\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{d\phi_2}{dt}&=\frac{\alpha}{2}\left(\Gamma G_n(N-N_{\rm tr})\left(1-\varepsilon_s A_2^2-\varepsilon_c A_1^2\right)-\frac{1}{\tau_{p2}}\right)\\&\quad-(\omega_{\rm ext}-\omega_{th})-\frac{K_{\rm ext}A_{\rm ext}}{A_2}\sin\phi_2\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{dN}{dt}&=\frac{\eta_i I}{eV}-\frac{N}{\tau_e}-G_n(N-N_{\rm tr})\\&\quad\cdot\left(\left(1-\varepsilon_s A_1^2-\varepsilon_c A_2^2\right)A_1^2+\left(1-\varepsilon_s A_2^2-\varepsilon_c A_1^2\right)A_2^2\right)\end{aligned} \tag{10}$$

where $G_n=v_g * g_n$. We have set the frequency of mode 2 to be the same as the external injected signal since there is little power in the initial nonlasing direction and finally it will be locked to the external source if locking conditions are satisfied. Although optical loss is related to the frequency of the mode and the losses for the two counter-propagating modes may be slightly different depending on the detuning frequency between mode 1 and the external injected source. However, they are assumed to be the same because the detuning involved is very small, therefore $\tau_{\rm p1}=\tau_{\rm p2}=\tau_{\rm p}$ in this paper. We also assume that the SRL is initially lasing in the CCW direction (mode 1).

B. Switching Conditions

As is well known that bistability can be achieved through gain saturation [5]–[7], and switching can be realized by external injection [7], but no specific switching conditions have been given. In this section we will discuss under what conditions the external optical continuous wave injected to the nonlasing direction can switch the lasing direction.

By deriving intensity equations with $dS/dt=dA^2/dt$ from amplitude rate equations, the rate equations without external injection are

$$\frac{dS_1}{dt}=\Gamma G_n(N-N_{\rm tr})S_1(\beta_1-\varepsilon_s S_1-\varepsilon_c S_2) \tag{11}$$

$$\frac{dS_2}{dt}=\Gamma G_n(N-N_{\rm tr})S_2(\beta_2-\varepsilon_s S_2-\varepsilon_c S_1) \tag{12}$$