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Passivity-based adaptive inventory control

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ABSTRACT

A new adaptive inventory control strategy is developed by applying online adaptation in the framework of passivity-based control. By using the system model and definition of the inventory, a feedback-feedforward control structure is derived from the passivity theorem. The stability analysis and the extension of the controller to a non-passive system are also given in this paper. This control strategy is demonstrated in a transfer function example and an application to a pressure tank unit in a chemical plant.

1. Introduction

The concept of passivity has played a key role of nonlinear control theory since it was formalized in 1970s [1]. Being related to Lyapunov and $\mathcal{L}_2$ stability, the passivity theory provides a useful tool for nonlinear system analysis [2]. An important conclusion from passivity is that a strictly passive system with a negative feedback of another passive system is stable [4]. Further more, the degree of passivity can be quantified by the passivity indices for both passive and non-passive systems. The shortage of passivity of a system can be compensated by the excess of passivity of a feedback controller, which motivated the passivity-based controller design [4].

Passivity theory has not been applied in process control until recent years. The notion of process system is first defined by connecting thermodynamics and passivity [5,6]. By using macroscopic balance of inventories (such as total mass and energy) to construct passive input-output pairs, an inventory control strategy is further proposed based on the idea that the manipulated variables are chosen so that the selected inventories follow their set points [7–9]. This control strategy has been applied to several control problem examples such as drum boiler, ternary flash and reactor flow sheet [7]. In industry, it was successfully applied to the temperature control problem of the float glass process for PPG company [9].

If there are unknown parameters in the process model, online adaptation can be used in the passivity-based inventory control frame [8]. In this article, the passivity-based adaptive inventory control is studied in both theory and applications, and the paper is organized as follows. In the following section, the notion of a passive system and the passivity theorem is summarized. The loop-shifting technique to render a non-passive system passive is also described. Then the adaptive feedforward and feedback controller is derived from passivity theorem and the inventory control theory. The Lyapunov stability analysis and extension to a non-passive system by loop-shifting are also given in this section. Finally, the control strategy is applied to a general transfer function example and a pressure tank unit of a chemical plant and simulation results are discussed.

2. Passive systems and the passivity theorem

In this section, let us review some of the basic definitions and theorems for passive systems. We will then use the loop-shifting technique to shift the passivity within the closed-loop to show how the stability is guaranteed based on passivity.

Definition 1 *(Linear Passive Systems [1]).* A linear time-invariant system Θ, with $n \times n$ transfer function matrix $T(s)$, is *passive* if and only if $T(s)$ is *Positive Real* (PR), or equivalently,

1. $T(s)$ is analytic in $\mathrm{Re}(s) > 0$
2. $T(j\omega) + T^{*}(j\omega) \geq 0$ for all frequency ω that $j\omega$ is not a pole of $T(s)$. If there are poles $p_1, p_2, \ldots, p_m$ of $T(s)$ on the imaginary axis, they are nonrepeated and the residue matrix at the poles $\lim_{s \to p_i}(s - p_i)T(s)$ $(i = 1, 2, \ldots, n)$ is Hermitian and positive semidefinite.

System Θ is said to be *strictly passive* or *S* trictly Positive Real (SPR) [3] if

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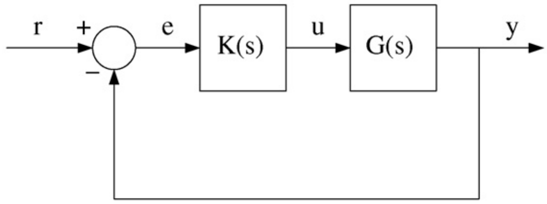

Fig. 1. Passivity theorem.

1. $T(s)$ is analytic in $\mathrm{Re}(s) \geq 0$
2. $T(j\omega) + T^*(j\omega) > 0$ for $\omega \in (-\infty, +\infty)$

Definition 2 *(Dissipative systems).* System Θ with supply rate $w(t)$ is said to be dissipative if there exists a non-negative real function $S(x) : X \to \mathbb{R}^+$, called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_0 \in X$ and $u \in U$,

$$S(x_1) - S(x_0) \leq \int_{t_0}^{t_1} w(t)dt, \tag{1}$$

where $x_1 = \phi(t_1, t_0, x_0, u)$ and $\mathbb{R}^+$ is a set of non-negative real numbers.

Definition 3 *(Excess/shortage of passivity).* Let $H : u \mapsto y$. System Θ is said to be:

1. Input feedforward passive (IFP) if it is dissipative with respect to supply rate $w(u, y) = y^T u - \nu u^T u$ for some $\nu \in \mathbb{R}$, denoted as IFP(ν).
2. Output feedback passive (OFP) if it is dissipative with respect to supply rate $w(u, y) = y^T u - \rho y^T y$ for some $\rho \in \mathbb{R}$, denoted as OFP(ρ).

A positive/negative value of ν or ρ means that the system has an excess/shortage of passivity.

Definition 4 *(Passivity Index).* The input feedforward passivity index for a stable linear system $G(s)$ is defined as

$$\nu_F(G(s), \omega) \triangleq \frac{1}{2}\lambda_{\min}(G(j\omega) + G^*(j\omega)) \tag{2}$$

where $\lambda_{\min}$ denotes the minimum eigenvalue.

For linear systems, the Passivity Theorem states that a feedback system (as shown in Fig. 1) comprised of a passive system $K(s)$ and a strictly passive system $G(s)$ is asymptotically stable.

If $G(s)$ in Fig. 1 is not passive ($\nu(G(s)) < 0$), then there exists $w(s)$ such that $G'(s) = G(s) + w(s)$ is passive, which means

$$\nu_F(w(s), \omega) + \nu_F(G(s), \omega) > 0 \quad \forall \omega \in \mathbb{R} \tag{3}$$

Figs. 1 and 2 are equivalent.

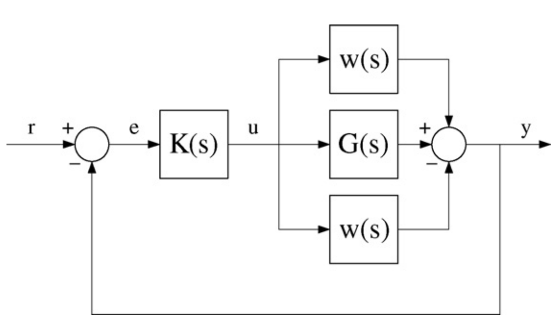

Fig. 2. Closed-loop modification.

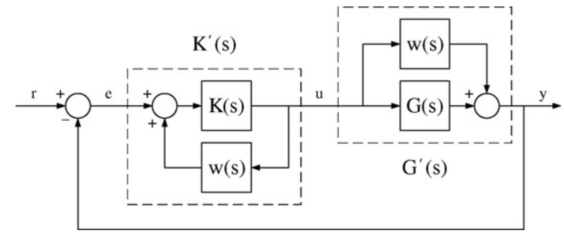

Fig. 3. Loop shifting.

However, since we can find $w(s)$ such that $G'(s) = G(s) + w(s)$ is strictly passive, we can shift the negative feedforward of $G(s)$ to become the positive feedback of $K(s)$ as shown in Fig. 3.

Hence, with $G'(s)$ being rendered strictly passive, we need to make sure the controller $K(s)$ with a positive feedback of $w(s)$, i.e., $K'(s) = [1 - K(s)w(s)]^{-1}K(s)$, is passive to guarantee the closed-loop stability based on the Passivity Theorem. In other words, G is IFP(ν) where $\nu < 0$ while K is OFP(ρ) where $\rho > 0$ and $\rho + \nu > 0$.

3. Adaptive inventory control

Consider a finite-dimensional dynamic system $\mathcal{S}$ represented by

$$\dot{x} = f(x) + g(x, u, d), \qquad x(0) = x_0 \tag{4}$$

$$y = h(x) \tag{5}$$

where x, u, d and y are the vectors of state variables, inputs and disturbances respectively; f, g and h are $\mathcal{C}^1$ functions.

An inventory for system $\mathcal{S}$ is defined by an additive continuous (C^1) function $Z : \mathcal{X} \to \mathcal{R}^+$ where $\mathcal{X}$ is the state space, so that the inventory of a system is equal to the sum of the inventories of its subsystems. Examples include total internal energy U, volume V and mass M (number of moles) of each components. From the continuity, the differential equality for the inventory vector Z can be written as

$$\frac{dZ(x)}{dt} = \phi(x, u, d) + p(x) \tag{6}$$

where

$$\phi(x, u, d) = \frac{\partial Z(x)}{\partial x} g(x, u, d) \tag{7}$$

$$p(x) = \frac{\partial Z(x)}{\partial x} f(x) \tag{8}$$

where ϕ is called the rate of supply, p is called the rate of production [7].

It is often possible to write

$$\phi(x, u, d) + p(x) = u\theta_0 + \mu(x, d)^T\theta \tag{9}$$

where $\mu(x, d)$ is a vector which depends on observable data; θ_0 is a non-zero parameter and θ is a vector of parameters which can be estimated on-line. Then Eq. (6) becomes

$$\frac{dZ}{dt} = u\theta_0 + \mu(x, d)^T\theta, \qquad \theta_0 \neq 0 \tag{10}$$

Let Z^* denote the reference of Z, then Eq. (10) can be rewritten as

$$\frac{d(Z^* - Z)}{dt} = \frac{dZ^*}{dt} - u\theta_0 - \mu(x, d)^T\theta \tag{11}$$

It can be proved that the input and output pair

$$U = \frac{dZ^*}{dt} - u\theta_0 - \mu(x, d)^T\theta \tag{12}$$

$$e = Z^* - Z \tag{13}$$