Probabilistic Load Flow Including Wind Power Generation

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***Abstract*—In this paper, a procedure is established for calculating the load flow probability density function in an electrical power network, taking into account the presence of wind power generation. The probability density function of the power injected in the network by a wind turbine is first obtained by utilizing a quadratic approximation of its power curve. With this model, the DC power flow of a network is calculated, considering the probabilistic nature of the power injected or consumed by the generators and the loads.**

***Index Terms*—Convolution, DC load flow, Fourier transform, power curves, probabilistic load flow, wind power, wind turbine.**

NOMENCLATURE

The following abbreviations will be used through the rest of the paper:

CDF	Cumulative Distribution Function
LF	Load Flow
MC	Monte Carlo
PDF	Probability Density Function
PLF	Probabilistic Load Flow
$\Pr(\cdot)$	Probability
pu	Per unit value
WF	Wind Farm
WT	Wind Turbine

I. INTRODUCTION

A. Probabilistic Load Flow

THE PLF consists of obtaining the system state and power flows in an electrical power network considering the probabilistic nature of the power injected by different kinds of generators, and bus loads. It was first proposed by Borkowska in 1974, developed by Allan [1]–[3], and further developed and applied into normal power system operation, short-term/long-term planning, as well as other areas.

In order to obtain system states and power flows in terms of PDF or CDF, the PLF requires the PDF or CDF of the input data to be known, so that system uncertainties can be included and reflected in the results. The PLF can be solved numerically, i.e., using an MC method, or analytically, i.e., using a convolution one, as well as a combination of these. The main concern about the MC method is the need for a large number of simulations, which is very time-consuming, whereas the main concerns about the analytical approach are the complexity of its mathematical developments and the possible lack of accuracy due to different approximations.

In this paper, a hybrid method is proposed, which includes wind power generation.

B. Wind Power Generation

The fact that wind power generation depends on a very variable resource, the wind, is well known. Wind variability can be observed spatially and temporally. The variation of wind speeds at a given location over time can be analyzed from a chronological point of view, which is important for generation forecasting, or in a cumulative way, which can be used for LF calculation. In this paper, we will focus on temporal variability of wind speeds from a cumulative point of view, making use of what can be considered their PDFs. Wind power integration in the PLF problem have been developed previously but not considering the quadratic model of the power curve of the WTs [4], [5].

C. Load Flow Solution

In order to obtain the solution of a power flow, the deterministic equations (1) and (2) have to be solved:

$$\mathrm{P_i} = \mathrm{V_i}\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{V_k}(\mathrm{G_{ik}}\cos\delta_{\mathrm{ik}} + \mathrm{B_{ik}}\sin\delta_{\mathrm{ik}}) \quad (1)$$

$$\mathrm{Q_i} = \mathrm{V_i}\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{V_k}(\mathrm{G_{ik}}\sin\delta_{\mathrm{ik}} - \mathrm{B_{ik}}\cos\delta_{\mathrm{ik}}) \quad (2)$$

where $\mathrm{P_i}$ and $\mathrm{Q_i}$ are the active and reactive powers being consumed by bus i, $\mathrm{V_i}$ and $\mathrm{V_k}$ are the rms values of voltages at bus i and neighbor buses k, δ_{ik} are the angles between voltages in buses i and k, and $\mathrm{G_{ik}}$ and $\mathrm{B_{ik}}$ the values of conductances and susceptances between them.

If the generated powers (except at the slack bus, whose voltage is generally used as an angle reference) and the bus loads in the system are all known, (1) and (2) can be solved for the bus voltages $\mathrm{V_i}$ and angles δ_{i}. From the solution, the active and reactive power flows through line ik, i.e., between buses i and k, are given by (3) and (4):

$$\mathrm{P_{ik}} = -\mathrm{t_{ik}}\mathrm{G_{ik}}\mathrm{V_i^2} + \mathrm{V_i}\mathrm{V_k}(\mathrm{G_{ik}}\cos\delta_{\mathrm{ik}} + \mathrm{B_{ik}}\sin\delta_{\mathrm{ik}}) \quad (3)$$

$$\mathrm{Q_{ik}} = \mathrm{t_{ik}}\mathrm{B_{ik}}\mathrm{V_i^2} - \mathrm{B'_{ik}}\mathrm{V_i^2} + \mathrm{V_i}\mathrm{V_k}(\mathrm{G_{ik}}\sin\delta_{\mathrm{ik}} - \mathrm{B_{ik}}\cos\delta_{\mathrm{ik}}) \quad (4)$$

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where t_{ik} are tap changer transformer ratios (this value is 1 in the case of lines or transformers without tap changers).

However, since (1) and (2) are nonlinear in voltage magnitudes and angles, the numerical solutions must be based on an iterative method, such as the Gauss-Seidel or the Newton-Raphson ones. But algorithms involving iterations can be lengthy, and in the probabilistic analysis, it is preferable to use linear approximations to (1) and (2), so that the state variables can be represented as linear combinations of input variables. This, in turn, will not only allow us to solve the LF equations through fast direct methods but will also permit application of convolution techniques to arrive at the probabilistic description of the variables of interest.

One of the approaches utilized to solve the LF equations by means of linear approximations is the DC LF, which has been described in [3] and [6].

D. DC Load Flow

Let us consider that in (1), we assume that $V_i = V_k = 1$ pu, $G_{ik} = 0$, and $\sin\delta_{ik} \approx \delta_{ik}$. Then, an equation for the power injected in bus i can be derived, which is (5):

$$P_i = \sum_k \frac{1}{X_{ik}} \delta_{ik} \tag{5}$$

where X_{ik} is the reactance of the line ik.

Of course, the powers of all the lines connecting buses k with bus i have to be added.

Equation (5) in matrix form becomes (6):

$$P = Y\delta \tag{6}$$

where Y is obtained through (7) and (8):

$$Y_{ik} = \frac{-1}{X_{ik}} \tag{7}$$

$$Y_{ii} = \sum_{i \neq k} \frac{1}{X_{ik}}. \tag{8}$$

In (6), the row and column corresponding to the slack bus are deleted. If Y is inverted, the angles δ are obtained, according to (9):

$$\delta = Y^{-1}P = \hat{Y}P \tag{9}$$

where the notation $\hat{Y}$ has been employed for the inverse matrix, in order to use the same notation for its elements.

From (3), the real power flow in the line ik becomes (10):

$$P_{ik} = \frac{\delta_i - \delta_k}{X_{ik}} = \sum_j H_{(ik)j} P_j \tag{10}$$

where $H_{(ik)j}$ is obtained in (11) and represents the contribution of bus j to the power through the line ik. If bus j is the slack one, then $H_{(ik)j} = 0$:

$$H_{(ik)j} = \frac{\hat{Y}_{ij} - \hat{Y}_{kj}}{X_{ik}} \tag{11}$$

where $\hat{Y}_{ij}$ and $\hat{Y}_{kj}$ are elements of the inverse matrix $\hat{Y}$, defined in (9).

Therefore, in order to obtain the real power flow through line ik, it is necessary to calculate the linear combination expressed in (10), where the elements $H_{(ik)j}$ depend on the configuration of the network.

The generated powers and bus loads can have probabilistic nature, so instead of utilizing deterministic values, their PDFs are used, also providing results in the shape of a PDF.

In order to obtain the PDF of the real power flow through line ik, P_{ik}, two types of operations have to be performed. The first one is the multiplication of a constant by a variable described by a PDF, which can be obtained by means of a change of variables, resulting in another PDF. The second one is the addition of variables depicted by their PDFs, which can be obtained utilizing the convolution of the PDFs, resulting in the desired PDF.

In Section II, a brief description of operations with PDFs is given.

II. Operating With PDFs

Operating with PDFs involves taking into account some considerations, such as explaining how to operate change of variables, convolution, and Fourier transform when PDFs have to be handled.

A. Change of Variables

If a variable y is a function of another variable x, that is, $y = g(x)$, then a change of variables is needed to obtain the PDF of variable y, taking into account that x has a known PDF. According to [7], when g is monotonic, and where $f_x(x)$ is the PDF of x, (12) can be applied, understanding that g^{-1} denotes inverse function and g' derivative one:

$$f_y(y) = \left| \frac{1}{g'\left(g^{-1}(y)\right)} \right| f_x\left(g^{-1}(y)\right). \tag{12}$$

If $y = ax$, where a is a constant value, as is the case in this analysis, the PDF of y is expressed as in (13):

$$f_y(y) = \left| \frac{1}{a} \right| f_x\left(\frac{y}{a}\right). \tag{13}$$

B. Convolution

If x and y represent two independent random variables with PDFs $f_x(x)$ and $f_y(y)$, respectively, and z is a new variable such that $z = x+y$, then the PDF $f_z(z)$ of z is obtained by convoluting $f_x(x)$ and $f_y(y)$, as expressed in (14):

$$f_z(z) = \int_{-\infty}^{\infty} f_x(x) f_y(z - x) dx. \tag{14}$$

C. Fourier Transform

When (14) cannot easily be solved, other techniques can be applied to obtain the convolution of two variables, x and y, with PDFs $f_x(x)$ and $f_y(y)$. The convolution is generally denoted by