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# On feasibility boundaries of electrical power grids in steady state

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ABSTRACT

Both the coordination of international energy transfer and the integration of a rapidly growing number of decentralized energy resources (DER) throughout most countries cause novel problems for avoiding voltage band violations and line overloads. Traditional approaches are typically based on global off-line scheduling under globally available information and rely on iterative procedures that can guarantee neither convergence nor execution time. In this paper, we focus on operational limitation problems in power grids based on widely dispersed (renewable) energy sources. We introduce an extension to the DEZENT algorithm, a multi-agent based coordination system for DER, that allows for the feasibility verification in constant and predetermined time. We give a numerical example showing the legitimacy of our approach and mention ongoing and future work regarding its implementation and utilization.


## 1. Introduction

The increasing integration of decentralized energy resources (DER) into distribution networks leads to new operational requirements in order to guarantee a secure and reliable energy supply. An increasing number of additional energy sources increases the complexity and amount of power flow distributions. In general this is taken into account by assigning the rated power of DER with respect to worst case power flow scenarios. This approach does not exploit the opportunities of increasing line usage rates by a coordinated operation of both, loads and DER on a particular network. One of the major issues related to this coordination is the recognition of critical operational states from a small set of information; in most cases the complex power balance of a connection point. Standard algorithms cannot be used in this time critical environment, as they cannot guarantee their convergence and execution time. In this paper, a new approach is presented that does have these properties and provides a coordinator with even more complex information than the standard algorithms do.

## 2. Previous and related work

In our earlier work on DEZENT [1–3] we introduced a bottom-up principle of power distribution and balancing, as part of a completely decentralized management of renewable electric energy production and consumption on the basis of real-time multi-agent systems. The completely decentralized approach adapts naturally to unpredictable situations by coordinating all agents in short time intervals of some milliseconds. This brings the need for a reliable and fast algorithm that allows for the determination of unfeasible network states. In Wedde et al. [4] we introduced the algorithmic basis for an integrated staged management of distributed electrical power grids based on the feasibility boundaries.

So far a lot of research has been conducted on determining the feasibility boundaries in the domain of nodal power. In Yuri et al. [5] the feasibility bounds in the body of active nodal power have been investigated. Lesieutre and Hiskens [6] analyzes the boundaries of feasible power flows and an analytical approach is presented to analyze their convexity properties. In Rote [7] the convexity of sets of feasible power injections is investigated and evaluated in terms of financial transmission rights. In this context we want to introduce an approach for the determination of the subspace of feasible combinations of nodal power in the domain of complex nodal power. Hereby we focus on distribution networks that have a single central feeder.

## 3. Notations and definitions

### 3.1. Variables and indices

| | |
|---|---|
| $\underline{A}$, $\underline{A}_i$ | vector or matrix and $i$th row vector |
| $\overline{A}_i$ | $i$th complex element of vector $\underline{A}$ |
| $\overline{Y}_{ij}$ | serial admittance between node $i$ and $j$ |
| $\overline{y}_{ij}$ | $(i,j)$-element of the nodal admittance matrix |

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*3.2. Orthonormal base vector sets*

$\underline{CS}(\underline{A})$ spanning the column space of matrix $\underline{A}$
$\underline{RS}(\underline{A})$ spanning the row space of matrix $\underline{A}$
$\underline{NS}(\underline{A})$ spanning the nullspace of matrix $\underline{A}$
$\underline{LN}(\underline{A})$ spanning the left nullspace of matrix $\underline{A}$

## 4. Geometrical interpretation of standard power flow equations

Traditionally the operational state of an electrical power grid is described using the vector of the complex-valued nodal voltages. Every other operational state value of the grid can be calculated from this voltage profile. This complex voltage profile is linked through different equations to the other domains, like e.g. by the power flow equations to the complex nodal power vector. The major drawback is that the complex nodal voltage profile is not available from measurement equipment, but has to be calculated from data, like complex nodal power, using iterative algorithms. These algorithms have an unpredictable convergence behavior and sometimes are even unable to find a solution at all. If there is a solution that was found, it is compared to operational constraints like voltage bands or maximum rated line currents. These algorithms are unable to provide any information about corrective measures in case of a constraint violation. As the convergence of the load flow calculation cannot be guaranteed, this class of algorithms are unsuitable for autonomous grid control.

In this article, the equations linking different domains will be analyzed for their mapping features between different vector spaces. The principle idea is to trace back how the voltage bands and maximum rated line current constraints are mapped to the domain of complex nodal power. It will be shown that this image can be calculated and described, so that it is possible to evaluate the feasibility of a certain complex nodal power combination right in the domain of complex nodal power. Furthermore, it is possible to calculate in a very illustrative way countermeasures in case an operational point violates constraints or is very close to do so.

*4.1. Properties of the power flow equations*

As the power flow Eq. (1) are complex-valued functions of complex numbers, they have to be complex differentiable in order to have a derivative.

$$\overline{S}_i = \overline{V}_i \cdot \sum_{j=1}^{n} \overline{y}_{ij}^* \cdot \overline{V}_j^* \tag{1}$$

In order to be complex differentiable all the partial derivatives of the initial function in real numbers have to exist, at least at the point of interest. For the power flow equations (1) their representation in real numbers are (2) and (3).

$$P_i\left(\underline{e},\underline{f}\right) = g_{ii}(e_i^2 + f_i^2) + \sum_{\substack{j=1\\ j\neq i}} (g_{ij}(e_ie_j + f_if_j) - b_{ij}(e_if_j - e_jf_i)) \tag{2}$$

$$Q_i\left(\underline{e},\underline{f}\right) = -b_{ii}(e_i^2 + f_i^2) - \sum_{\substack{j=1\\ j\neq i}}^{n} (g_{ij}(e_if_i - e_jf_i) + b_{ij}(e_ie_j + f_if_j)) \tag{3}$$

These partial derivatives are all defined for all points in $\mathbb{R}^{2n}$. Usually, they are arranged in the Jacobian matrix of the power flow equations. A complex derivative of the complex power flow equations only exists if the partial differential equations also fulfill the Cauchy–Riemann partial differential equations (4).

$$\frac{\partial P_i}{\partial e_j}\left(\underline{e},\underline{f}\right) = \frac{\partial Q_i}{\partial f_j}\left(\underline{e},\underline{f}\right); \quad \frac{\partial P_i}{\partial f_j}\left(\underline{e},\underline{f}\right) = -\frac{\partial Q_i}{\partial e_j}\left(\underline{e},\underline{f}\right) \tag{4}$$

A short observation shows that they are not fulfilled at any point except at zero nodal voltages. Thus, there is no complex derivative of the complex power flow equations. This means that the power flow equations are not analytic and cannot be fully treated by means of complex analysis. This poses a significant impact on the following analysis of the mapping properties of the power flow equations. Statements declaring the presence of certain properties like convexity of an image (a set of complex nodal power) cannot be derived from the properties of the preimage (a set of complex voltages).

*4.2. Constraints imposed by the voltage bands*

The presented algorithm is inspired by our findings that the voltage bands can be easily described in the domain of complex nodal voltage and the mapping of an operational point from the domain of complex nodal power to the domain of nodal voltages by the power flow calculation is not always possible.

In the following a way to map entire voltage bands to the domain of complex nodal power will be presented. By determining the image of the voltage band under the power flow equations it becomes possible to evaluate the feasibility of a certain nodal power combination, with respect to voltage band violations, by localization within the image of the voltage bands. If a power combination lies within the image it represents a power combination that does not lead to a voltage band violation.

In this subsection the general methodology for mapping the voltage band is presented. After a brief discussion about the form, the possibility to describe it and number of dimensions it will be shown how the mapping can be achieved and the resulting subspace in the domain of complex nodal power can be described.

While searching for voltage band violations of a given operational state of a network including a reference node denoted with index 1, the complex voltage of each node is evaluated against the following inequality

$$V_{min,i} \leqslant \left|\overline{V}_i\right| \leqslant V_{max,i} \forall i \in \{2,\ldots,n\} \tag{5}$$

The inequality (5) can be interpreted in a geometrical way. It describes a ring of feasible voltages in the complex plane of every nodal voltage like depicted in Fig. 1, except the voltage of the reference node which is fixed according to (6).

$$\overline{V}_1 = V_{ref} \cdot e^{j\cdot 0^\circ} \tag{6}$$

The ring in the complex plane of a node's voltage is not a convex set, but can be described using the two convex sets (7) and (8).

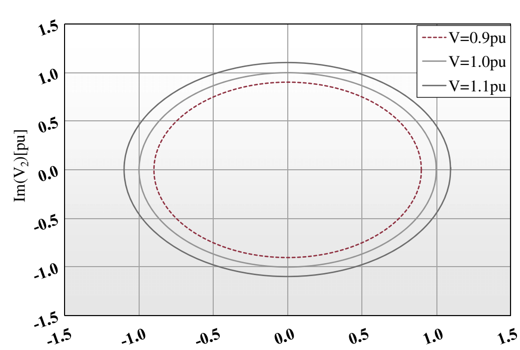


**Fig. 1.** Ring of feasible voltage.