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Single item inventory control under periodic review and a minimum order quantity

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ABSTRACT

In this paper we study a periodic review single item single stage inventory system with stochastic demand. In each time period the system must order none or at least as much as a minimum order quantity Q_{min}. Since the optimal structure of an ordering policy with a minimum order quantity is complicated, we propose an easy-to-use policy, which we call (R, S, Q_{min}) policy. Assuming linear holding and backorder costs we determine the optimal numerical value of the level S using a Markov Chain approach. In addition, we derive simple news-vendor-type inequalities for near-optimal policy parameters, which can easily be implemented within spreadsheet applications. In a numerical study we compare our policy with others and test the performance of the approximation for three different demand distributions: Poisson, negative binomial, and a discretized version of the gamma distribution. Given the simplicity of the policy and its cost performance as well as the excellent performance of the approximation we advocate the application of the (R, S, Q_{min}) policy in practice.
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1. Introduction

Single echelon single location inventory models have been extensively studied in literature (see for an overview Silver et al., 1998 or Zipkin, 2000). Assuming linear holding and penalty costs, and fixed reordering costs, the optimality of (s, S) and (R, s, S)-policies in continuous review and periodic review, respectively, is proven. Because of their simple structure, these policies are widely applied in practice and have been implemented in many business information systems, such as ERP and APS systems.

However, inventory managers in practice are sometimes confronted with additional constraints and requirements. As an example we mention the situation at a globally operating packaged goods company, where process efficiency demands that production batches are at least of a minimum size. Other examples can be found in apparel industries, where a minimum order quantity is not uncommon, too (see also Fisher and Raman, 1994; Robb and Silver, 1998).

The minimum order quantity restriction is not properly taken into account in the basic inventory models mentioned above. However, up to now little effort has been devoted to the modeling and analysis of inventory systems working with minimum order quantities. It has been proven that the optimal policy structure is complex (see Zhao and Katehakis, 2006) and typically complicated to implement in practice. Therefore, in literature the performance of different policy structures is investigated.

For low periodic demand relative to the minimum order quantity a mathematical model is presented in Robb and Silver (1998) to assist the decision maker when to order in case of a minimum order quantity. If the required amount is less than the minimum order quantity the actual ordersize can be increased or the order can be delayed. In a myopic approach both alternatives are compared in terms of costs in order to come up with formulae for the safety stock and the order threshold. In a large numerical study the authors show that their policy is outperforming a simple one, where the recommended order quantity is rounded up to the minimum amount.

Fisher and Raman (1994) have studied the stochastic inventory problem with a minimum order quantity for fashion goods. Since these products have very short life cycles with only few order opportunities, they investigate a two period model. They formulate a stochastic programming model to get insights in costs and the impact of the order constraint.

A two parameter policy, called (R, s, t, Q_{min}) policy, is studied in Zhou et al. (2007). It operates as follows. When the inventory position is lower than or equal to the reorder level s, an order is placed to raise the inventory position to $s+Q_{min}$. When the inventory position is above s but lower than threshold t, then exactly the required minimum amount is ordered. Otherwise no order is placed. In a numerical study the authors compare the proposed policy with the optimal one and conclude that the cost performance is close to optimal. However, to compute the cost optimal (R, s, t, Q_{min}) policy the steady-state probability

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distribution of the inventory position is needed and the authors claim themselves that *it is not clear how to calculate these steady state probabilities more efficiently than directly solving the linear system* associated with the balance equations. Thus, searching for the optimal policy parameters is computational intensive.

In this paper we propose a simple periodic review policy, called (R, S, Q_{min}) policy, where no order is placed as long as the inventory position, defined as the stock on-hand plus stock on-order minus backorders, is equal or larger than the level S. Otherwise an order is placed to raise the inventory to S. However, if this order is smaller than Q_{min} we increase the order quantity to Q_{min}. Note that this policy is a special case of the (R, s, t, Q_{min}) policy, viz. $s=S-Q_{min}$ and $t=S-1$. Formulating the associated Markov Chain model we can derive exact expressions for the holding and penalty costs for a given policy. This enables us to compute the optimal numerical value S^{opt} for each given Q_{min}. Since this procedure for finding S^{opt} is computationally intensive, we develop simple news-vendor-type inequalities from which a near optimal value S^*, can be routinely computed, e.g. using an EXCEL spreadsheet. In a detailed numerical study we compare the performance of the proposed policy with an optimal (R, s, t, Q_{min}) and an optimal (R, s, S) policy with $S-s=Q_{min}$. Moreover, the performance of our approximation is tested, yielding to excellent results. We conclude that the simplicity of the policy and the expressions for the computation of the policy parameter as well as cost performance of the (R, S, Q_{min}) policy justify an implementation in practice.

The remainder of the paper is organized as follows. In Section 2, the model and the notation is introduced. In Section 3 we first show how the optimal level S^{opt} can be computed and afterwards we develop the news-vendor-type inequalities mentioned above. In Section 4 an extensive numerical study is presented to test the performance of the policy and the approximation. Section 5 concludes the paper with a summary.

2. Model description

We consider a single item single echelon system with stochastic demand. In order to manage the inventory and place replenishment orders a periodic review system is used. We assume that the demand per period can be modeled with independent and identically distributed non-negative discrete random variables. Whenever demand cannot be satisfied directly from stock, demand is backordered. We further assume the length of the review period R to be given and without loss of generality we set it equal to one. Additionally, only order quantities of at least Q_{min} units are permitted and we assume the value of Q_{min} to be given. In order to determine replenishment times and quantities a so-called (R, S, Q_{min}) policy is applied. This policy operates as follows: at equidistant review timepoints the inventory position is monitored. If the inventory position is above the level S, then no order is triggered. In case the inventory position is below the level S, an amount is ordered which equals or exceeds Q_{min}. An amount larger than Q_{min} is only ordered, if the minimal ordersize Q_{min} is not enough to raise the inventory position up to level S (see Fig. 1 for an illustration of the policy).

The parameter S of the policy is therefore functioning as a reorder level as well as an order-up-to level. If the demand is always larger than the minimum order quantity, which may happen in case of small values of Q_{min}, then the order constraint is not restrictive anymore and the (R, S, Q_{min}) policy is similar to an order-up-to policy (R, S) with order-up-to level S. For large values of Q_{min} the parameter S functions as a reorder level only, and the policy is equal to an (R, s, Q_{min}) policy with a reorder level s.

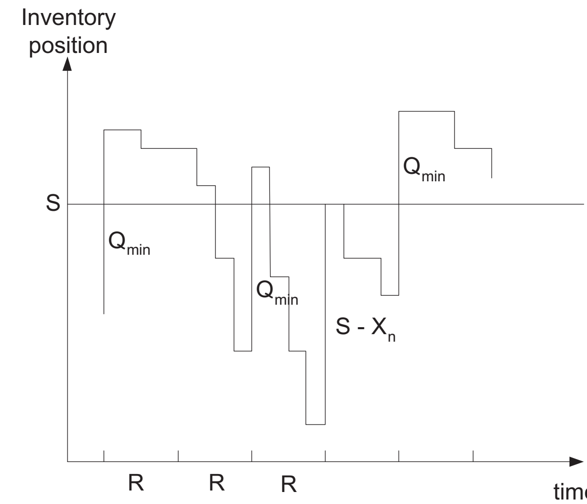

Fig. 1. The (R, S, Q_{min}) policy (leadtime equal to zero).

In order to evaluate the inventory system the average costs per review period are considered, composed of two main components. On the one hand the company incurs inventory holding costs and on the other hand backorder costs arise from stockouts. An inventory holding cost h is charged for each unit in stock at the end of a period and a penalty cost b is charged for each unit short at the end of a period. Note that fixed ordering costs are not included in the cost model.

The sequence of events is as follows. A possibly outstanding order arrives at the beginning of a period and the inventory position is reviewed and an order is placed if necessary. During the period, demand is realized and immediately satisfied if possible, otherwise demand is backlogged. Demand is satisfied according to a First-Come-First-Serve rule. At the end of the period holding and backorder costs are charged for each unit on stock or backordered.

The aim of the paper is to analyze the (R, S, Q_{min}) policy and determine an optimal level S^{opt} which minimizes the average holding and backorder costs per period in a stationary state, denoted as $C(S)$. Let I^+ and I^- denote the stock on hand and backlog at the end of a period. Thus the objective function can be written as

$$C(S)=hE[I^+]+bE[I^-] \tag{1}$$

In the remainder of this paper, the following notation will be used.

Q_{min}	minimum ordersize
S	policy parameter
L	leadtime
D_n	demand during the nth period
$D(i)$	demand during i periods
q_n	the quantity ordered at the beginning of the nth period
X_n	the inventory position before ordering, at the beginning of the nth period
Y_n	the inventory position after ordering, at the beginning of the nth period
I	inventory level at the end of a period
h	holding cost parameter per unit
b	backorder cost parameter per unit
$E[X]$	expectation of a random variable X
$\sigma(X)$	standard deviation of a random variable X
$c_v(X)$	coefficient of variation of a random variable X, ($c_v(X):=\sigma(X)/E[X]$)
X^+	$\max(0, X)$

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