Int. J. Production Economics 135 (2012) 116–124

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Inventory control in a two-level supply chain with risk pooling effect

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ARTICLE INFO

Article history:
Received 24 November 2009
Accepted 15 November 2010
Available online 23 November 2010

Keywords:
Supply chain
Inventory control
Risk pooling
Lagrangian relaxation
Heuristic

ABSTRACT

We consider an inventory control problem in a supply chain consisting of a single supplier, with a central distribution center (CDC) and multiple regional warehouses, and multiple retailers. We focus on the problem of selecting warehouses to be used among a set of candidate warehouses, assigning each retailer to one of the selected warehouses and determining replenishment plans for the warehouses and the retailers. For the problem with the objective of minimizing the sum of warehouse operation costs, inventory holding costs at the warehouses and the retailers, and transportation costs from the CDC to warehouses as well as from warehouses to retailers, we present a non-linear mixed integer programming model and develop a heuristic algorithm based on Lagrangian relaxation and subgradient optimization methods. A series of computational experiments on randomly generated test problems shows that the heuristic algorithm gives relatively good solutions in a reasonable computation time.

1. Introduction

We consider a two-level supply chain consisting of a single supplier and multiple retailers. In the supply chain, the supplier is composed of a central distribution center (CDC) and multiple candidate regional warehouses, from which up to a given number of warehouses are selected and actually used. It is assumed that the supplier is authorized to manage inventory levels of the retailers by a vendor-managed inventory (VMI) contract. In a VMI system, the supplier monitors inventory levels of the retailers as well as demands from final customers, and determines when and how much to deliver to the retailers as well as when and how much to replenish its own inventory at the warehouses. That is, the retailers do not place orders to the supplier, but the supplier controls the inventory levels of the retailers by determining replenishment timing and quantities for the retailers. It is known that by employing the VMI system, one can reduce the operating cost of the supply chain and maintain or improve the service level for the customers (Çetinkaya and Lee, 2000).

The problem considered here is to select warehouses to be actually used among a set of candidate warehouses, to assign each retailer to one of the selected warehouses, and to determine replenishment plans for the warehouses and the retailers, in the two-level supply chain that employs the VMI system. The warehouses and the retailers are assumed to use the (r, q) policy. That is, when the inventory level falls down to the reorder point, denoted by r, an order for q units is issued. Also, it is assumed that demands (per unit time) at the retailers (from final customers) are independent of each other and they follow normal distributions with mean m_j and variance v_j for retailer j, and that distances and lead times from CDC to warehouses as well as those from warehouses to retailers are known and fixed.

As the safety stock is generally set to be proportional to the standard deviation of the demand during the lead time, the safety stock can be reduced if the demand variation is reduced. Also, since demands from different retailers are independent, the variance of the sum of the demands from a set of retailers is smaller than the sum of the variances of the demands from those retailers. As a result, the safety stock needed for the pooled demands is generally less than the sum of the safety stocks for the individual demands. Therefore, to reduce operating costs of the supply chain, especially inventory holding costs, one may use the *risk pooling strategy*, the strategy of reducing the demand variability by aggregating demands from multiple retailers. Such aggregation can be done by allocating more retailers to each warehouse or reducing the number of warehouses to be selected and used.

In the example given in Fig. 1, which illustrates the supply chain considered in this study, safety stocks of warehouses 1 and 2 are set to $k^w\sqrt{(v_1+v_2)L_1}$ and $k^w\sqrt{(v_3+v_4)L_2}$, respectively. Here, L_i and k^w denote the lead time from CDC to warehouse i and the safety factor at the warehouses, respectively. Assume $L_1 \le L_2$. If we assign retailers 3 and 4 to warehouse 1 by not using warehouse 2, the safety stock of the supply chain can be reduced since the safety stock of warehouse 1 is set to $k^w\sqrt{(v_1+v_2+v_3+v_4)L_1}$, which is not greater than $k^w\sqrt{(v_1+v_2)L_1}+k^w\sqrt{(v_3+v_4)L_2}$.

The risk pooling strategy should be carefully applied since it may increase inventory levels of the retailers. As the number of warehouses that are selected and used is decreased, warehouse operation costs may be decreased and so may the safety stocks at the warehouses. However, lead times from the warehouses to the retailers may increase due to the increase of transportation distances

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doi:10.1016/j.ijpe.2010.11.014

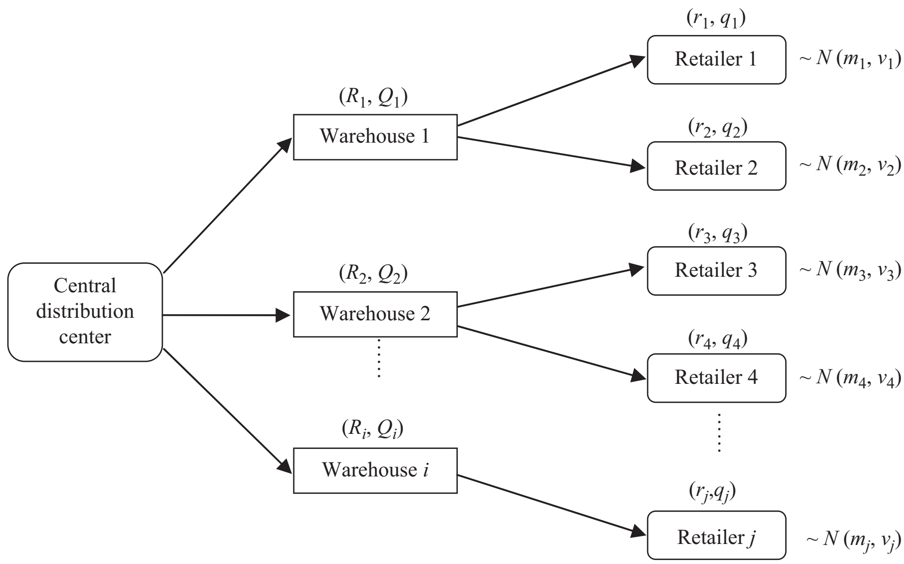

Fig. 1. Network diagram of the supply chain.

between the selected warehouses and the retailers, and hence the safety stocks at the retailers may be increased. Therefore, it is necessary to determine the optimal level of risk pooling, i.e., the optimal number of warehouses to be used, with the consideration of the trade-off between the decrease in the inventory holding costs at the warehouses and the warehouse operation costs and the increase in the inventory holding costs at the retailers.

Fry et al. (2001), De Toni and Zamolo (2005), Hong and Park (2006), Gumus et al. (2008), and Southard and Swenseth (2008) show benefits of a VMI system by comparing it with a traditional retailer-managed inventory system, and Wong et al. (2009) show that the performance of a supply chain can be improved through a sales rebate contract, which is devised to help centralize and coordinate decentralized decisions of a supply chain. Also, Szmerekovsky and Zhang (2008) investigated the effect of attaching radio frequency identification (RFID) tags at items on VMI system consisting of one manufacturer and one retailer, and Xu and Leung (2009) presented a stocking policy in VMI system with a limit on the shelf space. In addition, Çetinkaya and Lee (2000) and Axsäter (2001) presented analytical results for the coordination problem between inventory and transportation decisions in VMI systems, where a supplier has information on the demands from a group of retailers located in a given geographic region. On the other hand, Bertazzi et al. (2005) considered a production and distribution planning problem in VMI system and presented a decomposition approach to the problem, and Kang and Kim (2010) develop heuristic algorithms for an integrated inventory and transportation problem, in which a supplier determines the replenishment quantities and timing for retailers as well as the amount of products to be delivered by each vehicle with limited capacity.

As mentioned earlier, the benefit of risk pooling can be obtained through the consolidation of inventories of multiple locations into a single one. Eppen (1979), Chen and Lin (1989), and Chang and Lin (1991) show that a pooled system incurs less cost than a distributed system, and the difference of the costs of the two systems depends on the variance of demands and the correlation among the demands. Also, Alfaro and Corbett (2003), Gerchak and He (2003), and Benjaafar et al. (2005) investigated the benefits and costs of inventory pooling, and Kulkarni et al. (2005) evaluated trade-offs between logistics costs and risk pooling benefits in a manufacturing network with component commonality. In addition, Shen et al. (2003), Miranda and Garrido (2004), and Romeijn et al. (2007) used the risk pooling strategy in network design problems. However, their decisions are made only from the supplier's point of view since they do not consider the inventory holding costs at retailers. Also, without considering the inventory holding costs at the retailers, Miranda and Garrido (2004) show that as the inventory holding cost at warehouses, the variability of demands at retailers, and/or the service level increase the effect of risk pooling, i.e., cost reduction, increases. On the other hand, Gaur and Ravindran (2006) determined the best level of inventory aggregation for two conflicting objectives, maximizing responsiveness to customers' demands and minimizing the total cost of a supply chain, without considering inventory holding costs at the retailers.

As another alternative for reducing inventory of the supply chain, one can employ the policy of transshipment, i.e., replenishing inventories from locations at the same echelon level instead of a location at an upper level, since lead times can be reduced by employing the policy. Schwarz (1989) and Glasserman and Wang (1998) investigated the relationship between the lead time and the inventory level required for achieving a given service level for customers. Also, Grahovac and Chakravarty (2001) show that inventory sharing and lateral transshipment in a supply chain often reduce inventory holding costs and waiting costs of customers. In addition, Tagaras (1989), Archibald et al. (1997), Herer and Rashit (1999), Herer and Tzur (2001), Rudi et al. (2001), and Olsson (2009) dealt with transshipment problems in two-location inventory systems, while Tagaras (1999), Kukreja et al. (2001), Herer and Tzur (2003), Hu et al. (2005), Kukreja and Schmidt (2005), and Archibald (2007) developed inventory stocking policies in multiple-location inventory systems considering transshipments. Meanwhile, Lee (1987) and Axsäter (1990) presented lateral transshipment models for repairable items, and Evers (2001) and Minner et al. (2003) provided heuristic algorithms for determining transshipment timing.

In this study, we consider the problem of selecting warehouses from a given set of candidate warehouses, assigning retailers to the selected warehouses and determining replenishment plans at the warehouses and the retailers in a two-level supply chain, in which each member uses the (r, q) policy. We present a non-linear mixed integer programming model for the problem and develop a Lagrangian heuristic algorithm. In the next section, the problem considered in this study is described in more detail and a non-linear