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An analytical approach to the dynamic analysis of a rotating electric machine

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ABSTRACT

This study presents an analytical approach to the dynamic problem encountered by a rotating electric machine supported by nonlinear bearings with damping properties subjected to a forcing excitation caused by an unbalanced force and a parametric excitation caused by an axial thrust. The method employed in this analytical study is the Optimal Variational Iteration Method (OVIM), which proves to be very effective and accurate in solving nonlinear problems. The results are presented in the form of a time displacement response. The results obtained from numerical simulations are in very good agreement with the approximate analytical results obtained through the proposed method.

1. Introduction

Rotating electric machines are elastic systems exhibiting nonlinear vibration in their working regime. From the engineering point of view it is important to accurately predict the behaviour of such a system. This prediction is the key to the design of high-performance electric machines with higher speeds or longer periods between downtimes. The main causes that determine the occurrences of undesirable vibration in these dynamic systems are the nonlinear bearings, which support the rotating machine, the unbalanced forces, the mechanical looseness or misalignments. All these factors can affect the integrity of the system, therefore being highly detrimental [1,2].

The rotating electric machine under study is subjected to a parametric excitation caused by an axial thrust and a forcing excitation caused by an unbalanced force of the rotor while the entire system is being supported by nonlinear bearings with nonlinear stiffness characteristics and damping properties. This nonlinear suspension makes the analytical study very difficult, because it leads to strong nonlinear differential equations, which are hard to solve through classic methods. These kinds of dynamic problems are usually solved by numerical simulations [3], experimental investigations [4] and even by analytical developments [5,6].

Recently, new powerful analytical methods for nonlinear problems have been developed, such as the Adomian decomposition method [7], the modified Lindstedt–Poincare method [8], the parameter-expansion method [9,10], the homotopy perturbation method [11] etc.

The problem presented above for the considered rotating machinery can be modelled as a SDOF system which can be described by the second-order nonlinear differential equation

$$m\ddot{x} + c\dot{x} + k_1(1 - \lambda \sin \omega_2 t)x + k_2 x^3 = f \sin \omega_1 t \qquad (1)$$

where ω_1 and ω_2 are the forcing and the parametric frequencies and f and $k_1\lambda$ are amplitudes of the forcing and the parametric excitations, respectively. Therefore, the considered system will be characterized by two frequencies belonging to parametric and forcing excitations.

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For Eq. (1) we propose an approximate analytical solution using OVIM. This is an effective procedure for solving various nonlinear problems or differential equations with variable coefficients, without usual restrictive assumptions.

The Variational Iteration Method (VIM) was proposed by J.H. He in 1999 [12–15]. This method is now widely used by many researchers to study linear and nonlinear differential equations [16–25]. The method introduces a reliable and efficient process for a wide variety of scientific and engineering applications, linear or nonlinear, homogeneous or inhomogeneous equations and systems of equations as well. In case of VIM, initial approximations contain unknown parameters which can be identified by initial or boundary conditions after few iterations. In the case of OVIM, initial guess approximation contains more unknown parameters than boundary conditions. Some of these parameters can be identified from initial/boundary conditions and the rest of them can be identified optimally, so that the residual functional is minimized. In the case of OVIM we need only one iteration in order to solve the problem.

2. The optimal variational iteration method

We consider the following nonlinear equation:

$$\ddot{x} + \omega^2 x + f(t, x, \dot{x}, \ddot{x}) = 0 \tag{2}$$

where f is assumed to be a nonlinear function, which may be expanded in a Fourier series and $\dot{x} = \mathrm{d}x/\mathrm{d}t$. We construct the following iteration formula [13,14,16]:

$$x_{n+1}(t) = x_n(t) + \int_0^t \lambda(\tau, t)[x_n''(\tau) + \omega^2 x_n(\tau) + f(\tau, \tilde{x}_n, \tilde{x}_n', \tilde{x}_n'')]\mathrm{d}\tau \tag{3}$$

where $\lambda(\tau, t)$ is the Lagrange multiplier and can be identified via variational theory and $\tilde{x}_n$ is a restricted variation: $\delta x_n = 0$. After calculating variation with respect to x_n, we obtain the following stationary conditions:

$$\lambda''(\tau, t) + \omega^2 \lambda(\tau, t) = 0; \qquad \lambda(\tau, t)|_{\tau=t} = 0; \qquad 1 - \lambda'(\tau, t)\big|_{\tau=t} = 0. \tag{4}$$

The Lagrange multipliers, therefore, can be readily identified:

$$\lambda(\tau, t) = \frac{1}{\omega} \sin \omega(\tau - t) \tag{5}$$

and as a result, we obtain the following iteration formula

$$x_{n+1}(t) = x_n(t) + \frac{1}{\omega} \int_0^t \sin \omega(\tau - t) \left[x_n''(\tau) + \omega^2 x_n(\tau) + f(\tau, x_n, x_n', x_n'')\right] \mathrm{d}\tau. \tag{6}$$

Eq. (6) is equivalent to Eq. (7):

$$x_{n+1}(t) = x_0(t) + \frac{1}{\omega} \int_0^t \sin \omega(\tau - t) f(\tau, x_n, x', x'')\mathrm{d}\tau. \tag{7}$$

Initial conditions for Eq. (2) are:

$$x_0(0) = A, \qquad x_0'(0) = 0. \tag{8}$$

In our procedure, initial solution $x_0(t)$ contains $p > 2$ unknown parameters, which will be determined from Eq. (8) and the rest of $p - 2$ parameters can be optimally determined from the stationary conditions of the residual functional or by other methods such as Galerkin method, collocation method, least square method etc.

3. The analytical solution for the rotating electric machine vibration

We rewrite Eq. (1) as

$$\ddot{x} + \omega^2 x + (\Omega^2 - \omega^2)x + \mu \dot{x} - \alpha x \sin \omega_2 t + \beta x^3 - \gamma \sin \omega_1 t = 0 \tag{9}$$

with the initial conditions:

$$x(0) = A, \qquad \dot{x}(0) = 0 \tag{10}$$

where ω is the frequency of the system and:

$$\Omega^2 = \frac{k_1}{m}, \qquad \mu = \frac{c}{m}, \qquad \alpha = \frac{k_1 \lambda}{m}, \qquad \beta = \frac{k_2}{m}, \qquad \gamma = \frac{f}{m}. \tag{11}$$

As an initial guess for $x_0(t)$ we chose

$$x_0(t) = 2C_1 \cos \omega t + 2C_2 \sin \omega t + 2C_3 \cos 3\omega t + 2C_4 \sin 3\omega t \tag{12}$$

where C_1, C_2, C_3 and C_4 are unknown constants which can be partially determined from Eq. (10):

$$\begin{aligned} &2C_1 + 2C_3 = A \\ &C_2 + 3C_4 = 0. \end{aligned} \tag{13}$$