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A reference model for modal approximations of linear transmission line dynamics ⋆

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Abstract: Transmission line dynamics is an important subject for the simulation of fluid power systems. In the case of moderate amplitudes, linear models are widely applied. While the frequency domain offers a very compact and straightforward description of the input-output behaviour, the implementation of transmission line models in standard simulation software using a linear, time-invariant, finite-dimensional state space system description poses an approximation problem that has been treated by a large number of authors in the fluid power literature. This paper presents the solution of the approximation problem in an optimal way and gives a set of reference data.

Keywords: transmission line model, wave propagation, hydraulics

1. INTRODUCTION

Mathematical modeling towards simulation and control design for fluid power systems faces two major challenges. The first challenge is the abundance of nonlinearities present in hydaulic systems. In classical servo-hydraulics, the resistance controls are designed to operate with high flow velocity at the control edges of valves. This eliminates the influence of viscosity which is crucial given the tremendous variation of this material parameter over a typical operating temperature range. The predominance of convective acceleration over viscous friction leading to the neglibility of viscosity at the control input is however dearly payed for by the poor energetic efficiency of resistance control and by the quadratic coupling of pressure drop and flow rate. Another source of nonlinearities is the change of cylinder chamber volume with piston travel which gives rise to a shift of eigenfrequencies with cylinder position in a linearized model.

The second challenge is the interconnection of components like pumps, valves and actuators by possible long transmission lines. When the wave travel time across such a transmission line becomes comparable with characteristic frequencies of the interconnected system, wave propagation in the interconnection has to be accounted for. The characteristic frequencies can arise in the form of eigenfrequencies of certain system parts, they can be related to excitation frequencies or to the coupled overall system. In many cases, the behaviour of the transmission lines can be approximated by linear models.

The overall system model may now consist of a number of highly nonlinear sub-models interconnected by linear transmission line models. The input-output behaviour of these interconnections is easily described in the frequency domain by transcendental transfer functions, the time domain approximation using finite-dimensional state space models for use in standard simulation software is however not straightforward. This paper shows optimal results for one example. The goal is to provide a reference data set for comparison against existing methods.

2. AN EXAMPLE USING A CLASSICAL FREQUENCY DOMAIN MODEL

The transient flow of a weakly compressible liquid in a straight, rigid-walled transmission line of circular cross section can be desribed as in D'Souza and Oldenburger (1964)

$$\begin{bmatrix} \hat{Q}_0 \\ \hat{p}_1 \end{bmatrix} = \underbrace{\begin{bmatrix} \dfrac{\tanh(s\,Z)}{Z} & -\dfrac{1}{\cosh(s\,Z)} \\ \dfrac{1}{\cosh(s\,Z)} & Z\,\tanh(sZ) \end{bmatrix}}_{\mathbf{G}(s)} \begin{bmatrix} \hat{p}_0 \\ \hat{Q}_1 \end{bmatrix} \qquad (1)$$

with

$$Z = Z_0\sqrt{-\frac{J_0\left(\sqrt{-\frac{s}{\nu}}R\right)}{J_2\left(\sqrt{-\frac{s}{\nu}}R\right)}}, \gamma = \frac{sL}{c_0}\sqrt{-\frac{J_0\left(\sqrt{-\frac{s}{\nu}}R\right)}{J_2\left(\sqrt{-\frac{s}{\nu}}R\right)}}$$

and

$$Z_0 = \frac{\sqrt{K\rho}}{R^2\pi}, \quad c_0 = \sqrt{\frac{K}{\rho}}.$$

This model describes the input-output behaviour between the two ends of a line of length L and internal radius R

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Table 1. Parameter set.

L	5	m	length of line
R	$6 \cdot 10^{-3}$	m	internal radius of line
ν	$50 \cdot 10^{-6}$	$\frac{m^2}{s}$	kinematic viscosity
ρ	850	$\frac{kg}{m^3}$	mass denisty
K	$1.4 \cdot 10^9$	Pa	bulk modulus

filled with a liquid of density ρ, kinematic viscosity ν, and bulk modulus K. The pressure p_0 at one end and the flow rate Q_1 at the opposing end are taken as input variables, while the dual quantities Q_0 and p_1 serve as output signals. The parameters of the reference system treated in the present paper are given in Tab. 1. This case corresponds to a typical steel pipe of dimension 16x2 mm operated with a mineral oil based hydraulic fluid at room temperature.

3. $\mathcal{H}_2$-OPTIMAL TRANSMISSION LINE MODELLING

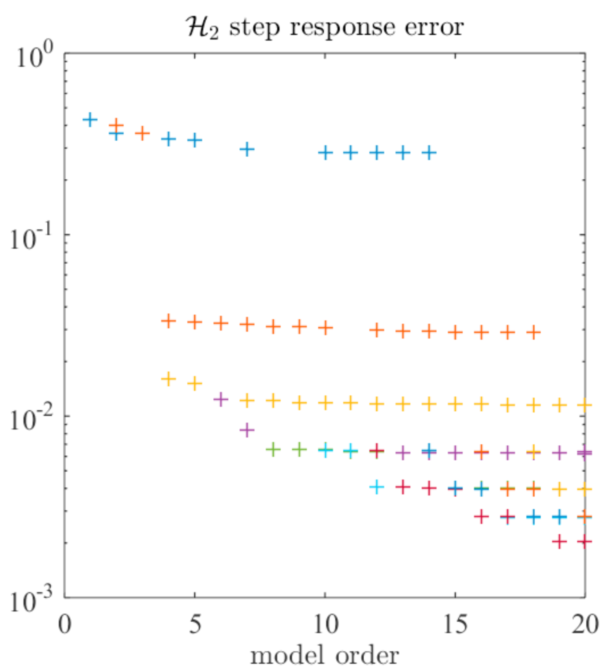

Fig. 1. $\mathcal{H}_2$-error norm according to eq. (6) over system order.

A time domain approximation of the model (1) can be derived in a large number of different ways. The first parting of these ways occurs when the decision between continuous and discrete time models is made. Discrete time models with a fixed time step are especially attractive in the method of characteristics approach, as for instance in Zielke (1968) and Suzuki et al. (1991). However, a fixed step, time-discrete wave propagation model disregards important coupling effects with nonlinear boundary conditions such as the switching of valves at arbitrary time instants or the dynamics of check valve models. Interpolation methods for overcoming the fixed step-size constraint at the boundaries of a method of characteristics model are possible, however in this approach the simplicity of the characteristical approach for constant wave speed problems gets compromised and the computational efficiency is questionable. Therefore, continuous time implementations are needed in a large number of simulation problems. These continuous-time models may exploit the knowledge about the wave travel time between the two boundary conditions, which gives rise to the transmission line method (TLM) modelling of wave propagation. The TLM needs an implementation of a pure time delay, which is not available in every simulation software package. If the implementation of the model is limited to linear, time invariant systems in state-space form like

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \begin{bmatrix} p_0 \\ Q_1 \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} Q_0 \\ p_1 \end{bmatrix} = \mathbf{C} \cdot \mathbf{x} + \mathbf{D} \cdot \begin{bmatrix} p_0 \\ Q_1 \end{bmatrix} \quad (3)$$

the approximation problem reduces to choosing a system order and finding the coefficients in the matrices **A**, **B**, **C** and **D**. A large number of approaches have been published in the fluid power literature. Modal approximations methods as suggested in Hullender and Healey (1981); Hullender et al. (1983) typically disregard important physical constraints like the passivity of the overall model or the limiting behaviour for zero and infinite frequencies. Method of lines discretizations using finite elements for the spatial discretization like Sanada et al. (1993) may also suffer from violations of the passivity constraints as shown and remedied in Manhartsgruber (2006a). A critical review of the abundance of available methods raises the question for a quality measure in the sense of a mathematical norm in order to compare the various methods and find an optimal solution. The problem with this optimization is obviously the need for constraints in order to exclude solutions that violate the underlying physics. The following approach has already been proposed in Manhartsgruber (2005, 2006c), however due to the high demand on computational resources the method was not investigated further. Now, after a decade of advance in computer power, the method is revisited in the present paper. The conditions for physical properness of the model and the optimization criterion are as follows:

3.1 Passivity constraint

The frequency domain counterpart of (2,3) is given as

$$\mathbf{G}_{approx}(s) = \mathbf{C}\left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} + \mathbf{D}. \quad (4)$$

With the assumption of all eigenvalues of **A** having strictly negative real part, the model is passive if and only if (see Khalil (1996))

$$\mathbf{G}_{approx}(j\omega) + \mathbf{G}^*_{approx}(j\omega) \geq \mathbf{0} \quad (5)$$

for all real ω. For the results presented in this paper, the passivity condition is enforced on a number of 1000 points distributed over the frequency range shown in Figs. 2 to 5.