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Investigation on the limitation of closed-form expressions for wideband modeling of overhead transmission lines

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ABSTRACT

Power system throughout the world is facing new challenges that may involve a wideband accuracy modeling of transmission lines. As high frequency phenomena are involved, displacement currents in the ground cease to be negligible and should be considered in the calculations. This work shows that the use of the so-called image approximations for the evaluation of the per-unit-length line parameters in multiphase transmission lines leads to unexpected numerical stability problems due to small passivity violations above a few MHz when short line span lengths need to be considered.

To illustrate that these violations are inherent to the image approximations, we considered three distinct line geometries commonly used in power systems involving different voltage levels. Several values for the ground parameters were also tested. The results indicate that passivity violations arise when image approximations are considered regardless of the line geometry. Furthermore, it was found that these passivity violations are related to the line length. In fact, there seems to be a minimal length suitable for the use of image approximations.

Time-domain responses based on the Numerical Laplace Transform (NLT), and the travelling wave method are presented. The input voltage in both approaches is perturbed by a small amplitude high frequency component to excite the passivity violations. Mitigation techniques to avoid unstable time-domain responses are discussed.

1. Introduction

An increasing need to improve the understanding and the evaluation performance of overhead transmission lines is required when wideband frequency phenomena are involved. Power line communication, lightning performance in lower ground conductivity areas, Gas-Insulated Substation (GIS) studies, secondary-arc modeling, interaction between overhead lines and Gas Insulated Lines (GIL) or underground cables and wide river crossing are just a few of the possible areas of interest where a wideband line model is sought. One consequence of this wideband modeling is the inclusion of the effect of ground permittivity in the per-unit-length impedance and admittance matrices. Rigorously, this could be achieved via an extension of a full-wave model for the general multi-phase case such as in [1,2]. However, this would require to iteratively solve a multi-dimensional integral equation. To avoid this cumbersome task, a common practice in power system is to adopt the quasi-TEM approximation neglecting ground displacement currents [3–5] or to seek even further simplification by using closed-form expressions based on image approximations [6–8] or other approximations of the infinite integrals terms involved [9–11]. There are some proposals for the inclusion of both ground displacement and conduction currents based on image approximations such as [12–15]. A recent work on lightning [16] indicates that the use of a more realistic current representation might lead to narrow overvoltage spikes of only a few μs and the results indicated that the ground permittivity was included. More recently, a distinct approximation has been proposed in [17] to allow for a wideband representation of impedance and admittance matrices.

Although some of these expressions have been proposed almost two decades ago, there has not been hitherto an assessment of the impact of these approximations in the numerical stability of a simulation carried out either in the frequency or time domains. In this work, we show that the use of image approximations in multiphase lines may lead to unstable time-domain simulations due to small passivity violations present above a few MHz in the high frequency range. Although these passivity violations might seem outside the usual bandwidth of interest for power system analysis, frequencies up to 10 MHz [18–21] or 100 MHz [22,23] might

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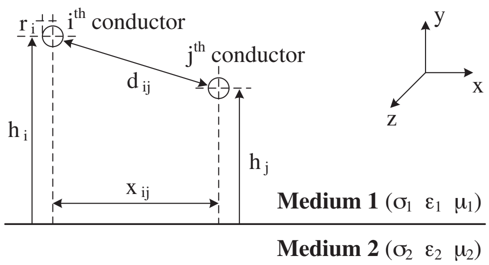

Fig. 1. Multiphase conductor arrangement.

need to be considered to improve the rational modeling of the propagation characteristic and the propagation function matrices. Furthermore, the correct assessment of the time-domain response of power line carrier channels such as the ones described in [24,25] might demand an accurate wideband representation of transmission lines.

2. Formulation of line parameters

A full-wave model overcomes the limitation regarding the behavior of the longitudinal component of the electric and magnetic field on the ground related to the quasi-TEM formulation. In this work, the expressions for the per-unit-length impedance and admittance of a single-phase line are based on a full-wave model originally developed by Kikuchi [2]. We could have adopted the full-wave model developed by Wait [1]. However, one advantage on the work of Kikuchi lies in the fact that it was developed using the electric scalar potential and the magnetic vector potential in which the formulation of the per unit length parameters is straight forward. Furthermore, as shown in [26], both Kikuchi [2] and Wait [1] present the same modal equation and then can be considered equivalent. For a single phase line, Pettersson [26] presented the expressions for the per unit length parameters for a wide frequency band using quasi-TEM and image approximations. Here, we formulate the corresponding expressions for the multi-phase case.

Consider a system of n infinitely long conductors with radius r at a constant height h_n above a lossy ground as shown in Fig. 1. Both air and ground are characterized by a permittivity ϵ_i, conductivity σ_i and permeability μ_i, where $i=1$ for air and $i=2$ for the ground, respectively. The propagation constant γ_i of each medium is given by

$$\gamma_i = \sqrt{j\omega\mu_i(\sigma_i + j\omega\epsilon_i)} \tag{1}$$

with the real part of the square root defined positive.

Each conductor has a voltage to ground U_n given by

$$U_n = V_n + j\omega \int_0^{h_n - r} A_y(0, \zeta)\, d\zeta \tag{2}$$

where $V_n = \varphi_n(0, h_n - r) - \varphi_n(0, 0)$ is the difference of the electric scalar potential $\varphi_n(x, y)$ between conductor and ground and A_y is the y-component of the magnetic vector potential $\mathbf{A}(x, y)$. See Appendix A for details.

The generalization of the quasi-TEM and image formulations for the general multi-phase case is presented next.

2.1. Quasi-TEM approximation

Assuming a thin wire approximation [10,27], quasi-TEM propagation, and ground and air permeabilities identical to the permeability of vacuum, i.e., $\mu_2 = \mu_1 = \mu_0$, the per-unit-length impedance and admittance matrices are then

$$\begin{aligned} \mathbf{Z} &= \mathbf{Z}_{\text{int}} + \frac{j\omega\mu_0}{2\pi}\left[\mathbf{P} + \mathbf{S}_1 - (\mathbf{S}_2 + \mathbf{T})\right] \\ \mathbf{Y} &= 2\pi\,(j\omega\epsilon_0)\left[\mathbf{P} - \mathbf{T}\right]^{-1} \end{aligned} \tag{3}$$

being $\mathbf{Z}_{\text{int}}$ the diagonal matrix with the internal impedance of each conductor. The matrix elements of $\mathbf{P}$ are given by

$$\mathbf{P}_{ii} = \ln\frac{2h_i}{r_i} \quad \mathbf{P}_{ij} = \ln\frac{D_{ij}}{d_{ij}} \tag{4}$$

with $D_{ij} = \sqrt{\ell_{ij}^2 + x_{ij}^2}$, $d_{ij} = \sqrt{(h_i - h_j)^2 + x_{ij}^2}$, $\ell_{ij} = h_i + h_j$ and x_{ij} is either the horizontal distance between conductors i and j or the conductor radius if $i=j$. The elements of $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{T}$ are given by

$$\begin{aligned} \mathbf{S}_{1_{ij}} &= 2\int_0^\infty \frac{e^{-\lambda\ell_{ij}}}{\lambda + \bar{u}}\cos(x_{ij}\lambda)d\lambda \quad \mathbf{S}_{2_{ij}} = 2\int_0^\infty \frac{e^{-\lambda\ell_{ij}}}{n^2\lambda + \bar{u}}\cos(x_{ij}\lambda)d\lambda \\ \mathbf{T}_{ij} &= 2\int_0^\infty \frac{\bar{u}}{\lambda}\left(\frac{e^{-\lambda\ell_{ij}/2} - e^{-\lambda\ell_{ij}}}{n^2\lambda + \bar{u}}\right)\cos(x_{ij}\lambda)\,d\lambda \end{aligned} \tag{5}$$

where $\bar{u} = \sqrt{\lambda^2 + \gamma_2^2 - \gamma_1^2}$ and $n = \gamma_2/\gamma_1$ is the refractive index of ground. If the ground is assumed to be a good conductor, i.e., $\sigma_2 \gg \omega\varepsilon_2$, it holds that the displacement currents can be neglected as both $\mathbf{S}_2$ and $\mathbf{T}$ tend to zero in (3). In this case, the resulting ground impedance is the same proposed by Carson in [3] where only $\mathbf{S}_1$ is considered.

2.2. Image approximation

In this approach, the quasi-TEM formulation can be further simplified to allow for a closed-form solution of the infinite integrals. In all the infinite integrals the following approximation is established

$$\left(a\lambda + \sqrt{\lambda^2 + \eta^2}\right)^{-1} \approx \frac{1}{\lambda}\frac{1}{1+a}\left[1 - \exp\left(-\lambda\frac{1+a}{\eta}\right)\right] \tag{6}$$

where $\eta = \sqrt{\gamma_2^2 - \gamma_1^2}$. Using this approximation with $a=1$ for $\mathbf{S}_1$ and $a=n^2$ for $\mathbf{S}_2$, it is possible to write

$$\begin{aligned} \bar{\mathbf{Z}} &= \mathbf{Z}_{\text{int}} + \frac{j\omega\mu_0}{2\pi}\left[\mathbf{P} + \bar{\mathbf{S}}_1 - \left(\bar{\mathbf{S}}_2 + \bar{\mathbf{T}}\right)\right] \\ \bar{\mathbf{Y}} &= 2\pi\,(j\omega\epsilon_0)\left[\mathbf{P} - \bar{\mathbf{T}}\right]^{-1}. \end{aligned} \tag{7}$$

The closed-form image approximation of the infinite integrals in (7) are given by

$$\bar{\mathbf{S}}_{1_{ij}} = \ln\left(1 + \frac{2}{\eta\sqrt{\ell_{ij}^2 + x_{ij}^2}}\right) \quad \bar{\mathbf{S}}_{2_{ij}} = \frac{2}{n^2+1}\ln\left(1 + \frac{n^2+1}{\eta\sqrt{\ell_{ij}^2 + x_{ij}^2}}\right)$$

$$\bar{\mathbf{T}}_{ij} = 2\ln 2 + \frac{2n^2}{n^2+1}\left(\ln\frac{1 + \frac{n^2+1}{\eta\sqrt{\ell_{ij}^2+x_{ij}^2}}}{1 + 2\frac{n^2+1}{\eta\sqrt{\ell_{ij}^2+x_{ij}^2}}}\right) \tag{8}$$

where the expression for $\bar{\mathbf{T}}$ presented here is slightly different from the one reported in [26].

3. Multiphase line model

Typically, the numerical stability of a multiphase transmission line model is verified using only its characteristic impedance $\mathbf{Z}_c$, i.e., a line model is assumed stable as long as $\mathbf{Z}_c$ has a positive real part. However, as presented next, this consideration alone might