# Beta (phase constant) compensation for flat voltage profile in a transmission line

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## ABSTRACT

The reliability of the bulk electric power systems is not only dependent on the generation but also on the transmission system. In long lines, the voltage along the transmission line varies with the loading conditions. Proper reactive power compensation helps to maintain the voltage along the line within acceptable limits. This paper proposes a compensation method to adjust the phase constant of the transmission line in order to obtain a flat voltage profile along the line. Case studies conducted on a long transmission line are presented.

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## 1. Introduction

The economic growth of any country depends on the availability of a quality electric power, one criterion of which is to maintain a constant voltage throughout the length of the line and making it flat. The major generating plants may not be near to the load centers and this condition dictates the power transmission over long distances. The bulk power transmission over longer distances is possible and economical if transmitted at higher voltages (EHV and UHV) [1]. The fundamental requirement for bulk transmission of electrical power is to keep the voltages near to its rated value and voltage variation along the line within acceptable limits. This is more important while transmitting at higher voltages in particular in order to keep the stress on the insulators below the rated value. One of the major concerns while transmitting power at high voltages is inadmissible increase of voltage along line on no load or low load conditions. Reactive power compensation provides voltage support and reduces the voltage fluctuation at a given terminal of a transmission line. It also improves the system stability by increasing the maximum active power that can be transmitted. It also maintains almost a flat voltage profile

along the transmission line [2]. If a lossless line is operated at its surge impedance loading, it exhibits a flat voltage profile. This has been discussed in detail in [3] controlling the surge impedance of the line by shunt compensation to make the corresponding surge impedance loading always equal to the actual loading. In this paper, a beta compensation (BC) method is proposed to implement the similar concept in a different way with a procedure to determine the required compensator values. This method requires a dynamic compensation and it can be achieved by using proper FACTS devices.

## 2. Long transmission line equivalent circuit

In a long transmission line model, sending-end voltage and receiving-end voltage can be related by

$$V_s = V_r \cosh \gamma l + I_r Z_c \sinh \gamma l \tag{1}$$

where $V_s$ is the sending-end voltage; $V_r$ is the receiving-end voltage; $I_r$ is the receiving-end current; $Z_c$ is the characteristic impedance, $\sqrt{z/y}$; $\gamma$ is the propagation constant, $\sqrt{yz} = \alpha + j\beta$; $\alpha$ is an attenuation constant, nepers/unit length; $\beta$ is the phase constant, radians/unit length; $l$ is the length of the line; $z$ is the series impedance/unit length and $y$ is the shunt admittance/unit length.

The voltage at any distance '$x$' from the receiving-end can be expressed as

$$V_x = V_r \cosh \gamma x + I_r Z_c \sinh \gamma x \tag{2}$$

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Tel.: +1 7327252556.
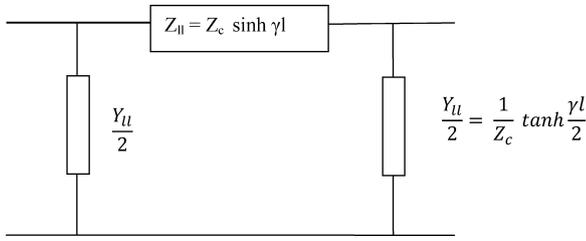*E-mail address:* nlsrao@yahoo.com (L.S.R. Nakka).

**Fig. 1.** Long line equivalent circuit.

By re-arranging the terms from Eqs. (1) and (2), the following equation can be derived

$$V_x = \frac{V_r \sinh \gamma(l-x) + V_s \sinh \gamma x}{\sinh \gamma l} \quad (3)$$

The lumped parameter equivalent circuit of a long line can be expressed as in Fig. 1.where $Z_{ll}$ is an equivalent long line series impedance and $Y_{ll}$ is an equivalent long line shunt admittance.

## 3. Voltage profiles

It is assumed in the work the line is lossless. This is of course a crude model far from reality. However, it is well known all initial generalizations are always made in the literature with respect to stability, economic operation of power systems and several optimization problems assuming the system is lossless. In this paper a new algorithm for providing a required compensation using the newly proposed Flatness Index is tested for implementation and hence to make possible approximate generalizations the line is assumed lossless. To make the line lossless, the series resistance and shunt conductance are neglected. Hence the attenuation constant ($\alpha$) becomes zero.

$$\gamma = \sqrt{yz} = \alpha + j\beta = j\beta$$

$$j\beta = \sqrt{j\omega C j\omega L}$$

$$\beta = \omega\sqrt{LC}$$

The quantity $\beta l$ is the electrical length of the line expressed in radians [4].

Very high voltages along the line may occur when $\beta l = n\pi$, where $n = 1, 2, 3\ldots$

For a loss-less line Eq. (3) becomes

$$V_x = \frac{V_r \sinh j\beta(l-x) + V_s \sinh j\beta x}{\sinh j\beta l} = \frac{V_r \sin \beta(l-x) + V_s \sin \beta x}{\sin \beta l} \quad (4)$$

With some arbitrary value of $\beta = 0.00205558\,\text{rad/mile}$ [5], $V_s = 1\angle 90°\,\text{pu}$, and $V_r = 1\angle 0\,\text{pu}$ the voltage profile along the long transmission line has been shown in Fig. 2.

From Eq. (4), the mid-point voltage, $V_m$ can be expressed as

$$V_m = \frac{V_r + V_s}{2\cos(\beta l/2)}$$

If $V_s = 1\angle\delta\,\text{pu}$, $V_r = 1\angle 0\,\text{pu}$, where $\delta$ is the power angle in radians.

$$V_m = \frac{\cos(\delta/2)}{\cos(\beta l/2)} \angle \frac{\delta}{2} \quad (5)$$

$$|V_m| = \frac{\cos(\delta/2)}{\cos(\beta l/2)} \quad (6)$$

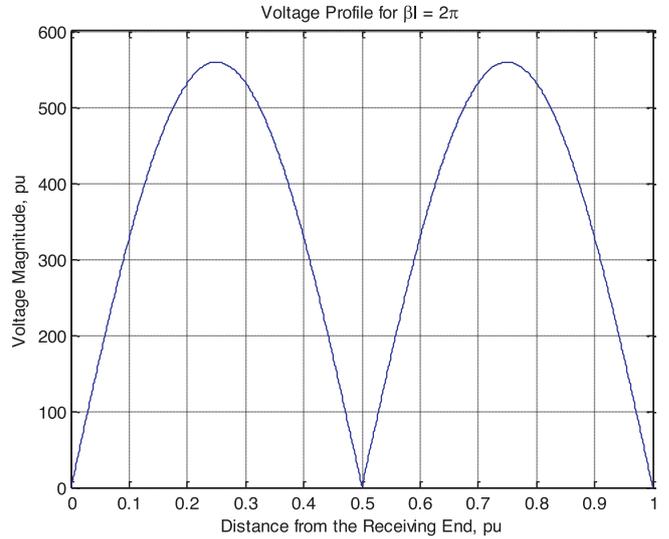Conditions for voltage sag, rise, and flat voltage profile:



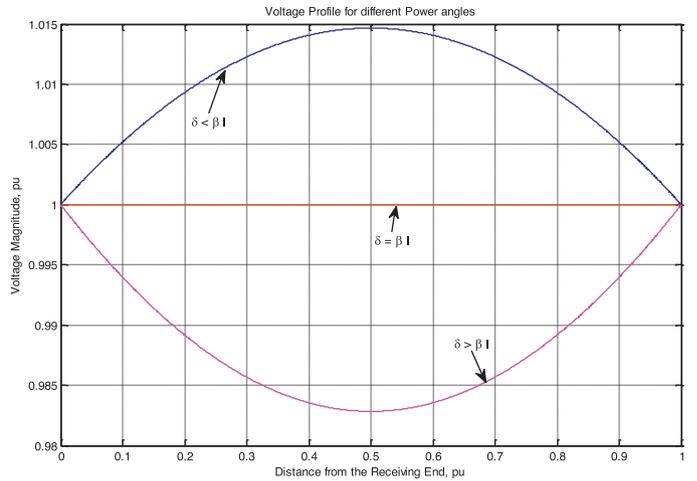**Fig. 2.** Voltage profile with $n = 2$.



**Fig. 3.** Voltage profile for different power angles.

$|V_m| < 1$, when $\delta > \beta l$, i.e. voltage sag
$|V_m| > 1$, when $\delta < \beta l$, i.e. voltage rise
$|V_m| = 1$, when $\delta = \beta l$, i.e. flat voltage

With the assumption that both $\delta$ and $\beta l$ vary between 0 to $\pi$ radians $(0 \le \delta \le \pi)$.

The following are the voltage profiles for these three different conditions.

## 4. Beta compensation (BC) method

From Eq. (6) and Fig. 3, it is evident that a flat voltage profile along the line can be obtained if the power angle ($\delta$) and the electrical length ($\beta l$) of the line are maintained at the same value. As explained earlier in Section 3, by neglecting the line series resistance and shunt conductance, the variation of $\beta$ with line series reactance and shunt susceptance can be shown as in Fig. 4.

With this keeping in mind, the following method has been proposed to maintain a flat voltage along the lossless long transmission line.

a. Load flow study is performed considering both ends of the line, one as slack bus and the other as PV bus (voltage controlled bus).
b. Power angle, the $\delta_{new}$ is compared with $\beta l$.